Five Inch Hollow Retroreflector

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Abstract: A five inch hollow cube has been proposed for a Lunar Rover. In principle, a cube of this size can provide a very high cross section so that a single cube is sufficient for laser ranging from earth. Large triangular cubes were used successfully on the Russian Lunokhod lunar rover. The problem with large cubes is thermal effects when the cubes are sunlit. Changes in the dihedral angles of hollow cubes with temperature have been seen over the range 4 - 10 arcsec. The cross section matrix for the cube has been computed for various dihedral angle offsets. The velocity aberration for Lunar ranging varies within a range of 3.5 - 7 microradians. If the dihedral angle offsets are larger than about 0.50 arcseconds the divergence of the reflected beam is larger than the Lunar velocity aberration. This results in very low cross section.

Contents:

Section 1. No dihedral angle offset.

Cross section matrix

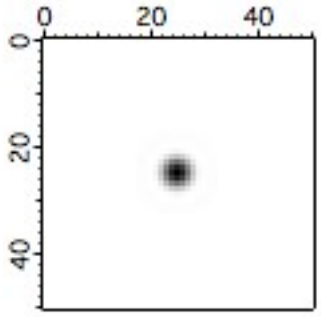

Figure 1a. Full scale

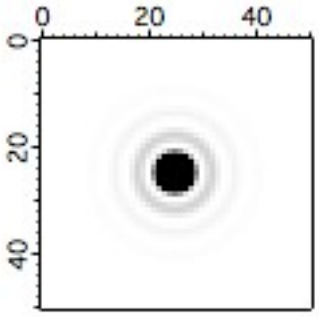

Figure 1b. 1/10 scale

Figure 1. Cross section matrix with no dihedral angle offset. The plot is a 51 x 51 matrix in each dimension. The plot is from -25 to +25 microradians in each dimension. Figure 1a is a plot of the full intensity. The pattern appears very narrow. Figure 1b is a plot with the values truncated at 1/10 the maximum. This plot shows the diffraction rings.

Table of cross section values

Vel. aberration	Cross section	
0.0000000	7124.876020	*
1.0000000	5956.169459	*
2.0000000	3836.799652	*
3.0000000	1702.809371	*
4.0000000	416.325742	*
5.0000000	33.311654	*
6.0000000	73.958358	*
7.0000000	114.352385	*
8.0000000	60.323819	*
9.0000000	8.953108	*
10.0000000	10.535582	*
11.0000000	26.666859	*
12.0000000	20.208137	*
13.0000000	4.656199	*
14.0000000	2.408773	*
15.0000000	9.194380	*
16.0000000	9.319051	*
17.0000000	2.889290	*
18.0000000	0.807019	*
19.0000000	3.757364	*
20.0000000	4.886890	*
21.0000000	2.089507	*
22.0000000	0.279978	*
23.0000000	1.771099	*
24.0000000	2.892507	*
25.0000000	1.613684	*

Table 1. Cross section (million sq m) vs velocity aberration (microradians). This is a combination table and graph. An '*' is used to plot the value on the printed page. The pattern is too narrow to cover the velocity aberration. The pattern has circular symmetry.

Section 2. Dihedral angle offset 0.50 arcseconds

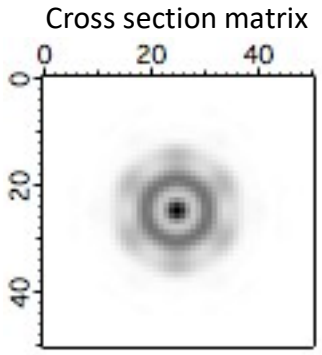

Figure 2. Cross section matrix with a 0.50 arcsecond dihedral angle offset in each of the three dihedral angles. The scale is -25 to +25 microradians in both directions.

Table of cross section values

Vel. aberration	Cross section
0.0000000	1031.324785
1.0000000	797.114053
2.0000000	447.478728
3.0000000	251.599735
4.0000000	325.213227
5.0000000	496.721645
6.0000000	541.081210
7.0000000	423.484385
8.0000000	278.963298
9.0000000	215.501258
10.0000000	207.911960
11.0000000	179.309418
12.0000000	111.540352
13.0000000	48.541472
14.0000000	21.669364
15.0000000	18.634460
16.0000000	14.721638
17.0000000	6.846868
18.0000000	5.040578
19.0000000	8.104785
20.0000000	8.484269
21.0000000	4.403061
22.0000000	1.608280
23.0000000	2.464205
24.0000000	3.180450
25.0000000	1.812657

Table 2. Cross section (million sq m) vs velocity aberration (microradians). The pattern does not have circular symmetry. The values are the average around a circle of radius given in the first column. The left and right parentheses show the minimum and maximum values around the circle. The dihedral angle offset increases the intensity on the first diffraction ring. The Lunar velocity aberration range is 3.5 to 7 microradians. The cross section in this range is up to 500 million sq meters.

3. Dihedral angle offset 1.0 arcseconds.

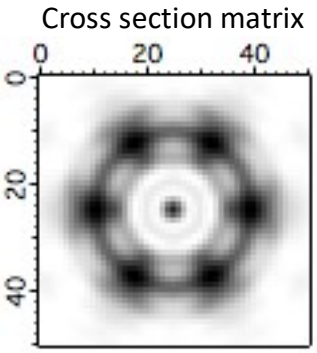

Figure 3. Cross section matrix with a 1.00 arcsecond dihedrla angle offset. The scale is -25 to +25 microradians in both directions.

Table of cross section values

Vel. aberration	Cross section
0.0000000	211.618447
1.0000000	153.886530
2.0000000	67.554693
3.0000000	13.523255
4.0000000	10.764068
5.0000000	20.155245
6.0000000	12.726358
7.0000000	7.270002
8.0000000	31.785610
9.0000000	75.441333
10.0000000	105.499308
11.0000000	114.786333
12.0000000	128.536187
13.0000000	160.275856
14.0000000	189.040377
15.0000000	185.950350
16.0000000	155.045557
17.0000000	120.102735
18.0000000	98.118345
19.0000000	80.784115
20.0000000	58.997192
21.0000000	35.334698
22.0000000	19.440549
23.0000000	12.504214
24.0000000	8.801447
25.0000000	5.510728

Table 3. Cross section (million sq m) vs velocity aberration (microradians). The cross section from 3.5 to 7 microradians is very low. The energy is concentrated on the third diffraction ring with six peaks around a circle. The cross section on the peaks is about 250 million sq m.

Section 4. Cross section for dihedral angle offset 2.0 arcseconds.

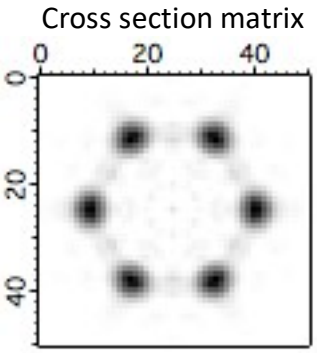

Figure 4. Cross section matrix with a 2.00 arcsecond dihedrla angle offset. The size of the pattern is larger than +/- 25 microradians. The scale is -50 to +50 microradians in both directions.

Table of cross section values

Vel. aberration	Cross section
0.0000000	23.067322
2.0000000	5.586604
4.0000000	1.573568
6.0000000	1.819709
8.0000000	3.513630
10.0000000	5.781922
12.0000000	2.211620
14.0000000	2.466149
16.0000000	4.997702
18.0000000	4.549310
20.0000000	4.343762
22.0000000	5.798189
24.0000000	17.388584
26.0000000	42.840643
28.0000000	62.392157
30.0000000	76.557603
32.0000000	71.038755
34.0000000	48.876585
36.0000000	29.559906
38.0000000	13.016613
40.0000000	6.485084
42.0000000	3.536746
44.0000000	2.339360
46.0000000	1.605527
48.0000000	1.437522
50.0000000	0.992972

Table 4. Cross section (million sq m) vs velocity aberration (microradians). The cross section from 3.5 to 7 microradians is extremely low. The energy is concentrated into six peaks around a circle. The intensity on the peaks is about 200 million sq m.

Section 5. Dihedral angle offsets all different (1.00,2.00,3.00)

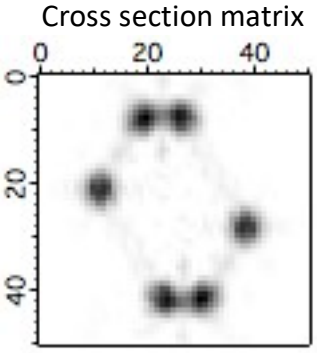

Figure 5. Cross section matrix with all dihedral angle offsets different (1.00,2.00,3.00) arcseconds. The shape of the pattern is asymmetric. The position of the peaks varies depending on the values of each of the 3 dihedral angle offsets. The scale is -50 to 50 microradians in both directions.

Table of cross section values

Vel. aberration	Cross section
0.0000000	1.926082
2.0000000	1.036541
4.0000000	3.186941
6.0000000	2.562618
8.0000000	2.644209
10.0000000	2.531442
12.0000000	2.780577
14.0000000	3.715670
16.0000000	4.327301
18.0000000	5.039758
20.0000000	8.913632
22.0000000	15.597051
24.0000000	21.642918
26.0000000	31.470137
28.0000000	38.978026
30.0000000	46.656591
32.0000000	50.114271
34.0000000	50.182475
36.0000000	44.347649
38.0000000	30.998953
40.0000000	18.348784
42.0000000	8.296338
44.0000000	3.957439
46.0000000	2.472528
48.0000000	2.112099
50.0000000	1.612583

Table 5. Cross section (million sq m) vs velocity aberration (microradians). The cross section from 3.5 to 7 microradians is extremely low. The cross section on the peaks is about 200 million sq m.

6. Single dihedral angle offset (2.00,0.00,0.00) arcseconds

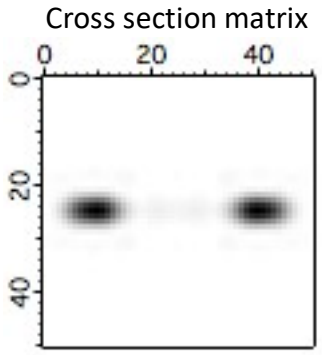

Figure 6. Cross section matrix with a single dihedral angle offset (2.00,0.00,0.00) arcseconds. The scale is -25 to +25 microradians in both directions.

Table of cross section values

Vel. aberration	Cross section
0.0000000	86.748167
1.0000000	77.892143
2.0000000	64.123497
3.0000000	52.852824
4.0000000	45.159093
5.0000000	35.676382
6.0000000	23.559800
7.0000000	16.401662
8.0000000	21.912119
9.0000000	40.468528
10.0000000	68.432678
11.0000000	101.662575
12.0000000	136.882480
13.0000000	167.279575
14.0000000	181.023041
15.0000000	175.247209
16.0000000	158.893394
17.0000000	138.471816
18.0000000	115.458263
19.0000000	89.262190
20.0000000	61.673999
21.0000000	37.824821
22.0000000	21.277073
23.0000000	11.608389
24.0000000	6.301786
25.0000000	3.802621

Table 6. Cross section (million sq m) vs velocity aberration (microradians). The cross section from 3.5 to 7 microradians is very low. The cross section is concentrated into two peaks aligned with the horizontal axis. The cross section on the peaks is very high, about 1700 million sq m.

7. Single dihedral angle offset (0.50,0.00,0.00) arcseconds

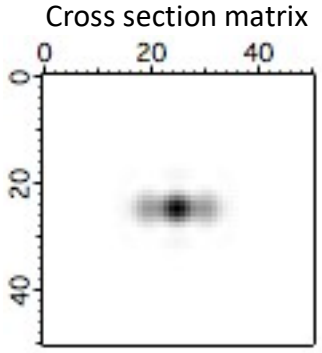

Figure 7a. Cross section matrix with a single dihedral angle offset (0.50,0.00,0.00) arcseconds. The scale is -25 to +25 microradians in both directions.

Cross section table

Vel. aberration	Cross section
0.0000000	3713.446058
1.0000000	3145.479997
2.0000000	2142.389023
3.0000000	1179.500517
4.0000000	639.691923
5.0000000	474.043744
6.0000000	411.919656
7.0000000	289.379449
8.0000000	142.904983
9.0000000	58.179742
10.0000000	40.943366
11.0000000	37.982562
12.0000000	21.026221
13.0000000	5.577534
14.0000000	5.818428
15.0000000	11.788414
16.0000000	10.021613
17.0000000	3.478122
18.0000000	2.229004
19.0000000	4.846846
20.0000000	4.998693
21.0000000	1.904514
22.0000000	0.579362
23.0000000	2.200154
24.0000000	2.972738
25.0000000	1.539254

Table 7a. Cross section (million sq m) vs velocity aberration (microradians). The cross section from 3.5 to 7 microradians is high. The cross section is concentrated into two peaks aligned with the horizontal axis.

Extended cross section table

Microrad	Minimum	Average	Maximum	Max – Min
0.0	3713.4460581	3713.4460581	3713.4460581	0.0000000
1.0	3081.4807734	3145.4799966	3319.7888434	238.3080700
2.0	1935.9131760	2142.3890235	2455.5732075	519.6600314
3.0	808.2646654	1179.5005170	1742.6458099	934.3811445
4.0	209.3100983	639.6919229	1499.1891743	1289.8790760
5.0	28.0590915	474.0437444	1520.0506538	1491.9915623
6.0	24.2860707	411.9196556	1415.6946851	1391.4086144
7.0	28.5300452	289.3794487	1049.6286489	1021.0986037
8.0	12.0677390	142.9049829	599.1962634	587.1285243
9.0	3.6677664	58.1797421	282.6205222	278.9527558
10.0	6.7214414	40.9433664	140.9026659	134.1812244
11.0	12.4375551	37.9825623	81.5156223	69.0780672
12.0	6.5752224	21.0262206	36.2055398	29.6303174
13.0	1.7828140	5.5775343	13.2175939	11.4347799
14.0	2.1275888	5.8184281	8.5981164	6.4705276
15.0	5.8555581	11.7884140	20.7923174	14.9367593
16.0	4.4216864	10.0216134	23.1404124	18.7187260
17.0	0.8404276	3.4781215	13.8528926	13.0124650
18.0	0.6762009	2.2290041	7.8405283	7.1643275
19.0	3.0585397	4.8468458	8.3514541	5.2929144
20.0	2.9728936	4.9986930	7.2203979	4.2475043
21.0	0.6956699	1.9045143	3.1645287	2.4688589
22.0	0.2491296	0.5793623	0.9954310	0.7463014
23.0	1.5394521	2.2001538	2.9355269	1.3960748
24.0	1.9316641	2.9727379	4.9663517	3.0346877
25.0	0.7969977	1.5392537	3.5785367	2.7815390

Table 7b. Minimum, average, maximum, and Max - Min around a circle in the far filed of radius given in the first column. The cross section on the two peaks is higher than in any of the other cases. The peaks would have to be aligned with the velocity aberration in order to take advantage of this very high cross section.

Cross section vs Velocity aberration

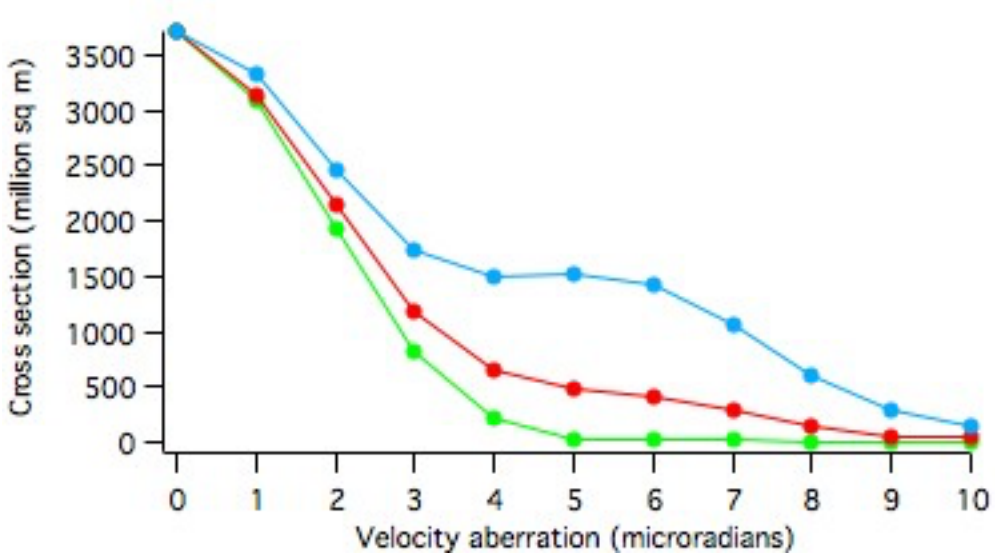

Figure 7b. Cross seciton (million sq m) vs Velocity aberration (microradians). The values are around a circle in the far field diffraction pattern. The cross section on the peaks (blue) is about 1500 milion sq m over most of the velocity aberration range 3.5 to 7 microradians. The peaks must be aligned with the direction of the velocity aberration to take advantage of the high cross section.

Green = minimum
Red = average
Blue = Maximum

8. Apollo 1.5 inch uncoated cube.

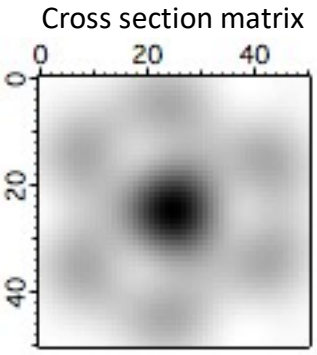

Figure 8. Cross section of a 1.5 inch uncoated cube.

Cross section table

0.0000000	14.323721	*
1.0000000	14.095344	*
2.0000000	13.570479	*)
3.0000000	12.743918	*)
4.0000000	11.679589	*)
5.0000000	10.452253	(*)
6.0000000	9.155824	(*)
7.0000000	7.869539	(*)
8.0000000	6.676529	(*)
9.0000000	5.638980	(*)
10.0000000	4.799783	(*)
11.0000000	4.179357	(*)
12.0000000	3.776612	(*)
13.0000000	3.567123	(*)
14.0000000	3.511236	(*)
15.0000000	3.565059	(*)
16.0000000	3.671634	(*)
17.0000000	3.783540	(*)
18.0000000	3.855402	(*)
19.0000000	3.857737	(*)
20.0000000	3.773509	(*)
21.0000000	3.597676	(*)
22.0000000	3.337190	(*)
23.0000000	3.009770	(*)
24.0000000	2.640600	(*)
25.0000000	2.255458	(*)

Table 8. Cross section (milion sq m) vs velocity aberration (microradians) of a 1.5 inch uncoated cube corner. The cross section varies from about 12 to 8 million million sq meters in the velocity aberration range from 3.5 to 7 microradians.

The cross section of a 5 inch cube with all dihedral angles set to 0.50 microradians is about 500 million sq meters. This is 500/10 = 50 times as large as a single Apollo cube corner. With a single dihedral angle offset the cross section is about 1500/10 = 150 times as large.

9. Variation of cross section with size of a hollow cube.

The thermal effects are a function of the size of the cube corner. The thermal effects can be reduced by reducing the size of the cube corner. However, this quickly destroys the advantage of the cube to concentrate the energy into a very narrow return beam. Figure 9 below plots the cross section on the central peak vs cube size.

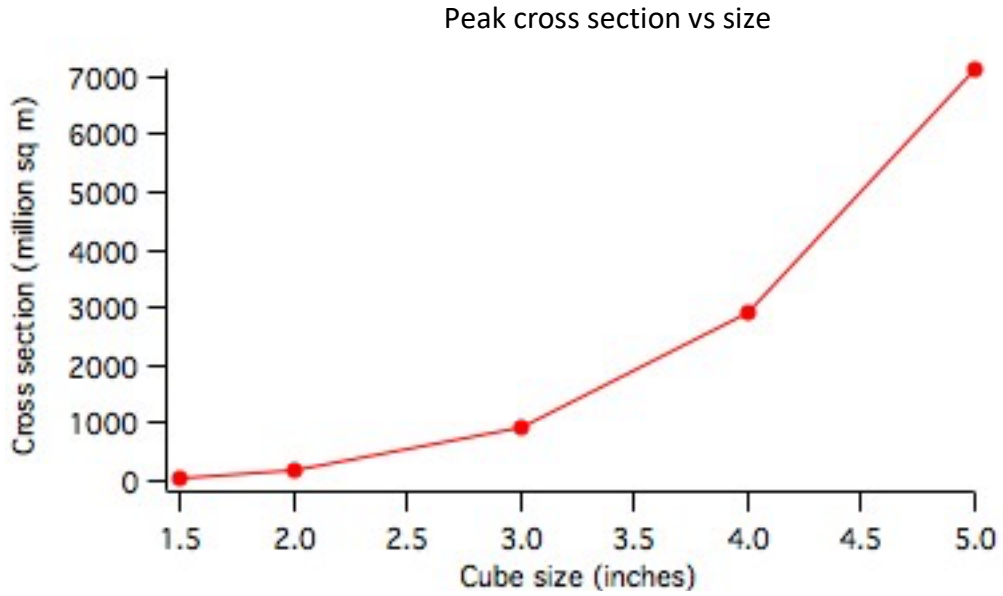

Figure 9. Peak cross section (million sq m) vs size (inches) of a hollow cube

Size	Cross section
5.00	7124.
4.00	2918.
3.00	923.
2.00	182.
1.50	57.

Table 9a. Peak cross section (million sq m) vs size (inches)

```
                                        Five inch
    0.0000000    7124.876020                                                  *
    1.0000000    5956.169459                                          (*)
    2.0000000    3836.799652                                  *)
    3.0000000    1702.809371                          (*
    4.0000000     416.325742   (*
    5.0000000      33.311654 *
    6.0000000      73.958358 (*
    7.0000000     114.352385  *
    8.0000000      60.323819 *
    9.0000000       8.953108 *
   10.0000000      10.535582 *
```

11.0000000	26.666859	*
12.0000000	20.208137	*
13.0000000	4.656199	*
14.0000000	2.408773	*
15.0000000	9.194380	*
16.0000000	9.319051	*
17.0000000	2.889290	*
18.0000000	0.807019	*
19.0000000	3.757364	*
20.0000000	4.886890	*
21.0000000	2.089507	*
22.0000000	0.279978	*
23.0000000	1.771099	*
24.0000000	2.892507	*
25.0000000	1.613684	*

Table 9b.

Four inch

0.0000000	2918.349218	*
1.0000000	2602.317666	(*)
2.0000000	1971.241100	(*
3.0000000	1205.072657	(*
4.0000000	556.154725	(*
5.0000000	163.569070	(*
6.0000000	18.206301	*
7.0000000	15.024943	*
8.0000000	43.038372	*
9.0000000	45.559546	*
10.0000000	24.814221	*
11.0000000	5.397861	*
12.0000000	1.519378	*
13.0000000	7.578495	*
14.0000000	11.557015	*
15.0000000	8.390426	*
16.0000000	2.730509	*
17.0000000	0.311317	*
18.0000000	2.092584	*
19.0000000	4.201903	*
20.0000000	3.887338	*
21.0000000	1.692998	*
22.0000000	0.196593	*
23.0000000	0.664137	*
24.0000000	1.829199	*
25.0000000	2.042546	*

Table 9c.

Three inch

0.0000000	923.383932	*
1.0000000	865.752177	*)
2.0000000	741.734715	*)
3.0000000	568.505780	*
4.0000000	383.410482	*
5.0000000	220.191954	*
6.0000000	101.305390	*)
7.0000000	31.947589	*
8.0000000	4.393784	*

9.0000000	2.377566
10.0000000	9.623063
11.0000000	15.141386
12.0000000	14.735976
13.0000000	9.835916
14.0000000	4.175909
15.0000000	0.778825
16.0000000	0.355277
17.0000000	1.843987
18.0000000	3.345298
19.0000000	3.647359
20.0000000	2.697212
21.0000000	1.247265
22.0000000	0.236014
23.0000000	0.123380
24.0000000	0.651697
25.0000000	1.242813

Table 9d.

Two inch

0.0000000	182.396826
1.0000000	177.248099
2.0000000	165.562111
3.0000000	147.567608
4.0000000	125.170307
5.0000000	100.519420
6.0000000	75.997580
7.0000000	53.444407
8.0000000	34.428405
9.0000000	19.751108
10.0000000	9.563142
11.0000000	3.472501
12.0000000	0.677254
13.0000000	0.138509
14.0000000	0.825155
15.0000000	1.911101
16.0000000	2.785718
17.0000000	3.151977
18.0000000	2.947094
19.0000000	2.341116
20.0000000	1.553935
21.0000000	0.810055
22.0000000	0.280031
23.0000000	0.039607
24.0000000	0.056339
25.0000000	0.242726

Table 9e

1.5 inch

0.0000000	57.711496
1.0000000	56.789497
2.0000000	54.657501
3.0000000	51.262346
4.0000000	46.814693
5.0000000	41.559784
6.0000000	35.826458

7.0000000	29.890112	*
8.0000000	24.065317	*
9.0000000	18.608590	*
10.0000000	13.731204	*
11.0000000	9.575802	*
12.0000000	6.224721	*
13.0000000	3.672488	*
14.0000000	1.880493	*
15.0000000	0.768355	*
16.0000000	0.189870	*
17.0000000	0.016968	*
18.0000000	0.113736	*
19.0000000	0.342730	*
20.0000000	0.605980	*
21.0000000	0.829851	*
22.0000000	0.967687	*
23.0000000	1.000747	*
24.0000000	0.937870	*
25.0000000	0.800213	*

Table 9f.

The peak cross section of a 1.5 inch uncoated cube corner is 14.323 million sq m. The peak for a coated cube is 57.711. There is a loss of aabout a factor of 4 in peak cross section by using an uncoated cube. The reason this was done for the Apollo cubes was to avoid the thermal problems caused by absorption of sunlight by the back faces of a coated cube.

10. Summary.

The isothermal cross section of a hollow cube can be as large as 500 - 1500 million sq meters. However, this requires very tight tolerances on the flatness and the dihedral angle offsets. The manufacturing tolerance of .25 arcseconds on the dihedral angles meets the requirement.

Changes with temperature in the range 4 - 10 arcseconds have been seen in the dihedral angle offsets. The Lunar velocity aberration is in the range 3.5 to 7 arcseconds. An offset more than about 0.50 arcseconds caused the reflected beam to split into separate beams whose divergence exceeds the velocity aberration. This results in very low cross section at the Lunar velocity aberration. The dihedral angle offsets should not exceed about 0.50 arcseconds.

My recollection is that the large cubes on the Russian Lunokhod Lunar Rover can only be used when they are not sunlit.

The Apollo retroreflectors use 1.5 inch uncoated cubes. The theoretical cross section is about 10 million sq m per cube. However, the actual cross section appears to be lower by a factor of as much as 30. The reason is unclear. The Apollo retroreflector arrays work when sunlit because of the low thermal effects.