USE OF A PASSIVE STABLE SATELLITE FOR EARTH-PHYSICS APPLICATIONS

Final Report

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Principal Investigator
Dr. George C. Weiffenbach

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Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

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1. INTRODUCTION

The mission of LAGEOS (LAser GEOdetic Satellite) is to make possible maximum-accuracy range measurements for both geometric and orbital-mode determinations of positions on the earth. The first spacecraft dedicated exclusively to laser ranging, it will provide the first opportunity to evaluate satellite laser ranging that is not degraded by errors originating in the target satellite.

The idea of orbiting a compact spherical satellite for laser ranging had been discussed at least as early as the first successful satellite laser observations, in 1964. At that time, it was known that substantial improvements in satellite-tracking accuracies would require some means of attenuating the effects of atmospheric drag and solar photon pressure; one obvious way to do this would be to use a very dense spherical satellite. Even in 1964, the accuracy of laser tracking instrumentation was high enough to make this concept very attractive.

A strong motivation for attaining orbit accuracies of 10 cm or better emerged from a seminar on Solid-Earth and Ocean Physics convened by the National Aeronautics and Space Administration (NASA) at Williamstown, Massachusetts, in August 1969 (Kaula, 1970). The geophysicists at this seminar suggested that satellite techniques be applied to the measurement of crustal motions, both on a global scale and in somewhat more detail in active fault zones. It was stated that, with sufficient accuracy, this new information would have a profound effect on our knowledge of solid-earth dynamics and on earthquake research.

In 1970, Smithsonian Astrophysical Observatory (SAO) started an examination of how such a satellite might be configured. This study was accelerated when Dr. John De Noyer (at that time, the NASA Headquarters Director of Earth Observation Programs) informed SAO that a launch-vehicle test flight of a Saturn IB rocket was under consideration and that a heavy passive satellite would be an ideal hitchhiker for this launch. Accordingly, SAO designed a 76-cm (30-inch) diameter, 3600-kg (8000-lb) satellite configured to the Saturn IB capability. Not surprisingly, this satellite acquired the name "Cannonball," even though the Air Force had already launched several satellites (for atmospheric probing) with this same name.

SAO submitted a proposal to NASA for a Phase I study of Cannonball in October 1970,* for which this document is the final report. A Cannonball briefing was given by The Manned Space Flight Center (MSFC) and SAO to the NASA administrator, Dr. James C. Fletcher, in March 1971. At that time, NASA decided that the Cannonball launch could be justified only if it were incorporated in a formal NASA program. When the Saturn test launch did not materialize, the Cannonball configuration was shelved.

During this same period, a program plan for an Earth and Ocean Physics Applications Program (EOPAP)† was being developed at NASA Headquarters (SAO participated in this activity). The Cannonball concept is retained in EOPAP, but the satellite name has been changed to LAGEOS and a Delta vehicle is now specified.

SAO has completed its design studies for the Delta-launched LAGEOS, and the final spacecraft configuration is defined in this report, which was delayed for that express purpose. The LAGEOS configuration described here is optimum, in that it has the maximum mass-to-area ratio feasible with the specified launch vehicle but is still large enough to allow initial orbit acquisition with the Baker-Nunn cameras and subsequent tracking by existing laser systems.

The two salient characteristics of LAGEOS are the accuracies it will make possible – viz., ±2-cm station-position accuracy – and its great simplicity and ease of fabrication. Table 1 describes the final LAGEOS configuration.

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*Proposed to NASA for Using a Passive Stable Satellite for Earth-Physics Applications (Cannonball), SAO Proposal P 277-10-70.

†Earth and Ocean Physics Applications Program, NASA Headquarters, September 1972.

Table 1. LAGEOS Parameters.

Satellite Configuration	
Shape:	Sphere
Radius:	22 cm (17.3-inch diameter)
Mass:	680 kg (1500 lb)
Mass-to-area ratio:	4470 kg m^{-2}
Exterior surface (excluding retroreflectors):	Aluminum, diffuse at thermal wavelengths (~ 10 μm)
Material:	Depleted uranium (U^{238})
Retroreflector Cube Corners	
Circular front face:	3.65-cm diameter
High-purity fused silica	
Dihedral angle:	90° + 1.75 ± 0.5 arcsec
No reflective coatings	
No antirelection coatings	
Total number:	240
Orbit Parameters	
Nodal period:	166 ± 2 min
Inclination:	90° ± 1°
Eccentricity:	0.020 ± 0.015
Nominal altitude:	3700 km (2000 nm)

The satellite design is now complete. Detailed optical, mechanical, and thermal analyses have been performed, together with comprehensive studies of the influence of various spacecraft parameters on the range measurements. LAGEOS could be ready for launch within 12 months. This short lead time is based on the fact that LAGEOS is actually quite an easy satellite to build, for several reasons: standard machining and assembly tolerances exceed the needed accuracies; the cube-corner specifications are well within the state of the art; and the satellite is passive, has a small number of components and no moving parts, and is made of simple and available materials.

LAGEOS will make available for the foreseeable future an in-orbit capability for laser ranging of maximum accuracy. The high mass-to-area ratio and the precise, stable (attitude-independent) geometry of the spacecraft, in concert with the proposed orbit, will make this satellite the most precise position reference available. Because it will be visible in all parts of the world and will have an extended operating life in orbit, LAGEOS can serve as a fundamental global standard for decades. It will constitute an important first step in the EOPAP.

2. LAGEOS MISSION

EOPAP requires a satellite range-measuring accuracy of 2 cm, in accord with one of the principal recommendations of the Williamstown study (Kaula, 1970): that NASA develop techniques for obtaining relative positions of points on the earth to that accuracy. A similar recommendation was made by the Space Science Board (1971): that solid-earth physics "would require location accuracies on the order of ± 2 cm in a program lasting decades... ."

Range measurements with 2-cm accuracy will be used to accomplish many of the EOPAP objectives, such as the determination of plate tectonic motions, regional fault motions, the rotation and wobble of the earth, and earth-body tides. These objectives must be attained by measuring the variations with time of the internal geometry of a global matrix of fiducial points on the earth's surface, of the fiducial points with respect to the earth's center of mass, and of the matrix with respect to an inertial reference. These kinematic variations are known to have time scales ranging from a day (e.g., body tides) to millenia (e.g., continental drift).

What is needed, then, is a means for making exceedingly accurate measurements on a global basis in such a way that first, each position on the globe can be related to all others and to the earth's center of mass; second, complete sets of observations can be obtained in less than a day; and third, continuity of observations is maintained over the longest possible time span. The first two considerations clearly suggest the use of a satellite in a high-inclination orbit; the third suggests that the satellite be passive. A satellite fitted with laser retroreflectors is an appropriate choice. The 1967-68 National Academy of Sciences (NAS) summer study recommended that such a satellite be included in plans for the United States space effort (Doyle, 1969) (the NAS panel discussed this satellite under the heading GEDY-4).

A satellite that is optimum for EOPAP kinematic measurements should have the following characteristics:

A. Such a satellite should have the maximum feasible mass-to-area ratio in order to reduce perturbations caused by nongravitational forces (mainly radiation pressure).

B. The satellite should be compact and rigid for maximum stability of spacecraft geometry.

C. It should be spherical so that the geometry of the retroreflector array versus the spacecraft center of mass will not change with aspect. The spherical shape is also necessary to minimize errors in computing corrections for radiation pressure and drag.

D. A completely passive satellite is desirable in order to attain maximum operating life. The satellite will be acquired by camera (by photographing reflected sunlight against star background) and will be equipped with retroreflectors for ranging with ground-based lasers.

E. The orbital altitude should be high enough to reduce to an acceptable level orbit errors resulting from uncertainties in geopotential models.

F. Its orbital altitude should be low enough to provide good signal-to-noise ratios with a retroreflector array of reasonable dimensions.

G. The inclination of the satellite should be high enough to provide global coverage.

3. LAGEOS ACCURACY REQUIREMENTS

Consideration of the EOPAP objectives clearly substantiates the recommendations of the Williamstown study and the Space Science Board for accuracies of ± 2 cm. The simple fact that secular motions as slow as 1 cm yr^{-1} are important to the program in itself confirms this need. The facts that each ground station in the program will be subject to the complex motions of earth rotation, polar wander, tectonic and fault motions, etc., and that the motions resulting from each phenomenon must somehow be sorted out before the observations can be fully exploited, emphasize further the need for this level of accuracy.

One of the more important factors in designing LAGEOS is the error that the satellite itself is allowed to contribute to the 2-cm total of the laser observations. To establish a design goal for this satellite error, we must estimate the magnitudes of the errors contributed by other sources – that is, we must formulate an error budget that anticipates the future state of the art in laser ranging. Although predictions of this kind are always somewhat uncertain, we believe the following are reasonable expectations for 5 to 10 yr from now:

Tropospheric-propagation velocity uncertainties	15 mm
Laser	
Pulse detection	10 mm
Range counter	5 mm
Cables, mechanical, calibration-target survey, calibration propagation velocity, etc.	5 mm
Epoch (time synchronization)	5 mm
Satellite	5 mm
Root sum square	20 mm

The above values are based on an assumption that some portion of the errors is uncorrelated, or not systematic over time intervals of the order of a LAGEOS pass, so that this portion can be reduced by aggregating some number of pulses. For example, a purely random range-counter error of $\pm$ 0.1 nsec can be reduced to $0.1 \times 1/\sqrt{100} = 0.01$ nsec = 0.15 cm by averaging over 100 pulses.

4. LAGEOS ORBIT

In the calculation of station positions for the 1969 Smithsonian Standard Earth (II) (SE II) (Gaposchkin and Lambeck, 1970), geometric and dynamic solutions were used in combination because the combined solution was superior to that obtained from either method by itself. The use of both techniques is of further importance because a comparison of the results obtained independently by each method provides a unique check of accuracy. In addition, we should like to exploit both the accuracy of geometric solutions and the inherent (earth) center-of-mass coordinates of the dynamic method. We have applied this dual approach to the choice of orbit and to the design of the satellite.

The central problem in designing LAGEOS is to find the best compromise among three competing factors: orbital altitude, mass-to-area ratio (and, therefore, payload weight), and launch-vehicle capability. The LAGEOS mission will require orbit determination to unprecedented accuracy, which will be achieved only by making a strenuous effort to control orbit perturbations. In terms of satellite design, the latter can be accomplished by three means: adjusting the orbit (primarily the satellite altitude), reducing the satellite accelerations produced by surface forces by increasing the mass-to-area ratio, and configuring the satellite to improve the accuracy to which perturbations can be computed (spherical shape and stable surface characteristics).

Some, though not all, orbit perturbations can be reduced by increasing the orbit altitude, as discussed in detail in Sections 4.2 and 4.3. However, all the perturbations except the gravitational one can be reduced by increasing the mass-to-area ratio, which suggests lowering the orbit altitude to allow more satellite weight. Also, the return-signal strength is strongly attenuated by increasing range (R^{-4}), which implies that the orbital altitude should not be any higher than necessary. Furthermore, the rate at which information is generated usually increases as the mean motion – and therefore the number of passes per day – increases, which also suggests a lower altitude.

The proposed satellite size, mass, and orbit are believed to be the best compromise among these several conflicting factors; a compromise that is also compatible with the capability of the launch vehicle now assigned to LAGEOS. We will discuss the various factors that impinge on the choice of orbit and present estimates of the orbit errors that result from uncertainties in the forces acting on LAGEOS.

4.1 Orbit Requirements for Geometric Position Determination

Geometric solutions are determined through trigonometric calculations based on simultaneous observations of a satellite from two or more ground stations. Geometric solutions with range measurements are sometimes thought to require simultaneous observations from four ground stations, but this is not necessary if a sequence of measurements is made over a common satellite arc from each of two stations. Although the observed segment of the satellite orbit must be used in computing relative station positions, orbit errors have very little influence on the computed positions because they are common to the observations from each station. An analogous approach has been used successfully for several years with the TRANSIT system, which utilizes range-difference observations. Error amplification in computing station positions from a single satellite pass observed from two stations will generally be unacceptable. However, this impediment can be removed by using two or more such passes with differing geometries in each determination. This requirement is quite compatible with the EOPAP mission criteria. We can also include in this category quasi-simultaneous observations by using short arcs of the satellite trajectory if the arcs either partially overlap or are close enough that orbital errors do not significantly influence the results. This independence of orbital error and the attendant independence of geophysical assumptions constitute the main advantages of this method. In addition, the direct geometric approach is conceptually straightforward, and the computations are quite reliable.

One obvious requirement for any orbit (for both geometric and dynamic methods) is that it must be visible from all observing sites of interest. EOPAP must include all land areas. For reasons of accuracy, we should stipulate further that the minimum elevation angle used in observing the satellite should be 15°. If we assume that approximately 50 global sites might be occupied during the course of EOPAP, the average separation of adjacent sites would be just under 30° (great circle), suggesting a satellite altitude of about 3000 km. A ground-station network for the EOPAP cannot, of course, be uniformly distributed over the earth, and even if, as seems probable, the number of sites exceeds 50, there will be some instances where the separation may exceed 60° – e.g., in connecting the southern tip of Africa with that of South America. However, since the proposed satellite will provide very accurate solutions for station positions when used in the dynamic or the orbital mode, it is not essential that we provide for simultaneous observations in every case.

We conclude that a satellite altitude 3000 km or higher should be suitable.

4.2 Orbit Requirements for (Dynamic) Orbital Position Determination

In dynamic solutions, separate observations made from sites in all geographic areas within view of the orbit are related by orbital mechanics, so there is no necessity for simultaneous observations. Thus, we can relate the positions of ground stations with any geographic separation without the need for imposing mutual-visibility requirements on the orbit. Since station locations are calculated with respect to the orbit, they are automatically determined in earth center-of-mass coordinates. When combined with a geometric solution, the dynamic solution will also control and limit the error amplification inherent in the step-by-step extension of a geometric net.

An orbiting satellite intrinsically defines an inertial system. It is coupled to the earth through the earth's gravity field and, to a much lesser degree, through the earth's atmosphere; but with an appropriate choice of orbit, the influence of coupling of the orbit can be calculated to rather high accuracy, particularly for orbital arcs less than 30 days or so. This inertial character of the satellite enables us to use dynamic solutions to determine rigid-body motions of the earth, such as rotation (UT1) and polar motion. The principle has already been demonstrated by Anderle and Beuglass (1970) in their determination of polar motion from satellite doppler measurements. It should be noted that geometric solutions provide no information on these phenomena, since the internal geometry of the ground-station network is invariant under rotation and translation.

Since the satellite orbit is the connecting link in relating station positions to each other and to an inertial frame, uncertainties in orbit computation are propagated into the dynamic solutions. [Lambeck (1971) has examined the effects of orbital errors and data accuracy on the determination of polar motion through laser observations of satellites, including LAGEOS.] This point has been the most important consideration in our choice of orbit and spacecraft design. Two factors must be considered: the accuracy to which the forces acting on the satellite can be calculated, and the extent to which orbit errors impede the filtering processes used to elicit the parameters of interest, such as when characteristic periods of the errors match those of the parameters. The orbit parameters that can be adjusted to control these effects are primarily altitude and inclination.

The orbital altitude provides the greatest degree of control. The three forces that significantly influence satellite trajectories are gravity, atmospheric drag, and photon pressure. (Only uncertainties in the earth's gravity field need be examined because the gravitational forces exerted by other bodies – sun, moon, and planets – can be calculated a priori to sufficient accuracy.) Orbital errors arising from gravity and drag can be reduced by increasing satellite altitude. Errors arising from photon pressure can be reduced by increasing the mass-to-area ratio.

Two aspects might be considered in an examination of orbital errors caused by inaccuracies in our knowledge of the earth's gravity field: the absolute value of the field and its structure. The former is contained in the constant GM. Errors in GM are decoupled from and have no practical influence on the accuracy to which other earth-physics parameters can be determined.

The structure of the gravity field raises more complex problems. A basic tenet is that the orbital altitude must be adjusted in order to avoid, as much as possible, all resonances with the geopotential, both because the magnitudes of the physical perturbations are very much larger under resonant conditions and because, in order to minimize problems of aliasing, we must suppress perturbations that have periods commensurate with the earth's rotation. Thus, we should avoid satellite altitudes that result in mean motions of exactly n or n + 1/2 revolutions per day, etc.

Aside from resonant effects, orbit perturbations caused by geopotential structure are attenuated by increasing satellite altitude. This is a selective process, in that the effects of short-wavelength features in the geopotential fall off more rapidly with altitude than do those of long-wavelength terms. We have calculated the magnitudes of these effects for each term in an expansion of the geopotential in spherical harmonics up to 20, 20. The results are expressed in terms of sensitivity coefficients (or influence coefficients) for each harmonic of degree ℓ and order m.

The most effective touchstone for estimating geopotential effects is a comparison of the sensitivity coefficients for a projected orbit with those for an orbit that has been tracked to some known accuracy. We have selected the GEOS 1 orbit as a standard, since it has been one of the most intensely and most accurately tracked satellites.

On the basis of extensive computations, we estimate that the GEOS 1 orbit error resulting from geopotential errors in SE II does not exceed 10 m. For example, in carefully tracked orbital arcs up to 4 weeks in length, it has been possible to fit laser range data to rms residuals of about 7 m. Using the sensitivity coefficients to extrapolate to the proposed LAGEOS orbit, we estimate that the contribution of the geopotential error to this orbit should not exceed 50 cm (based on SE II).

The 5-cm accuracy requirement cannot be met for the proposed LAGEOS orbit without improvement in the accuracy of gravity-field models. There is no question of the feasibility of the needed accuracy improvement, and indeed the 10-cm geoid required in EOPAP for other applications will more than satisfy the needs of LAGEOS.

We conclude that, with the improvements in the geopotential planned as part of EOPAP, (geopotential) orbit errors for LAGEOS will be reduced to the required levels for orbits at altitudes of 3700 km or higher. The proposed orbital parameters for LAGEOS follow:

$$T = 166 \text{ min} = 1/8.65 \text{ sidereal day} \quad ,$$

$$i = 90° \quad ,$$

$$e = 0.02 \quad .$$

These parameters result in an average altitude of roughly 3650 km.

Other orbital altitudes have been considered for LAGEOS. Table 2 lists some that satisfy the condition that resonances be avoided. Also included are the payload weights for the TAT(9C)/Delta/TE 364 launch vehicle at each altitude and the relative amplitudes of the orbit perturbations caused by direct solar and earthshine photon pressure. These relative amplitudes, normalized to unity for the proposed LAGEOS orbit, have been derived under the assumption that the spacecraft size is adjusted for each orbit to correct for the R^{-4} range attenuation, so that the same echo strength is obtained in each orbit at 30° elevation.

Table 2. Alternative LAGEOS orbits and payload weights for the TAT(9C)/Delta/TE 364 launch vehicle in polar orbits.

Orbit (rev/sidereal day)	Orbit altitude (km)	Payload weight (kg)	Relative magnitudes of orbit perturbations	
			Direct solar	Earthshine
8.55	3720 ←	680	1.0	1.0
7.55	4600 ←	600	2.9	1.9
6.55	5690 ←	500	9.3	3.8
5.55	7100	440	33	7.9
4.55	9000	390	120	14.8
3.55	11800	320	600	32

Since orbital errors caused by inaccuracies in the geopotential model should be reduced to acceptable levels for an orbital altitude of 3700 km during the course of EOPAP, there appears to be no reason to go to a higher orbit, where the more intractable photon-pressure effects are sharply increased.

In addition, greater mean motion at lower orbital altitudes results in more satellite passes per day at each station. Since each pass is an independent data set, better results are obtained with more passes. For example, with the proposed orbit, a midlatitude station will have an average of six passes with 20° or higher elevation angles each day, with both northbound and southbound (satellite motion) passes, with passes both to the east and to the west of the station, and with a variety of elevation angles. This variation in pass geometry will significantly reduce the influence of orbital errors.

4.3 Nongravitational Orbit Perturbations

The nongravitational forces acting on the spacecraft are photon pressure, atmospheric drag, and micrometeorite impacts, in order of decreasing magnitude. Interactions of the paramagnetic body of the satellite with the earth's magnetic field, and the effects of the accumulation of electric charge on the satellite, have been calculated and are not significant.

The disturbing force f produced by incident photons on a satellite with cross section area A_c is

$$f = KA_c \frac{I}{c} \quad ,$$

where K is a constant that depends on the reflecting properties of the surface, and I is the intensity of the incident light. For total absorption or specular reflection, K = 1.0; for a diffuse surface, K = 1.44. We will set K = 1.22 for our satellite since the surface is roughly half diffuse metal and half cube corners.

The intensity of direct sunlight – the solar constant – is known to about 1%. We will adopt the value (from Drummond, 1970)

$$I = 135.7 \text{ mw cm}^{-2} = 1.357 \times 10^6 \text{ ergs sec}^{-1} \text{ cm}^{-2} \quad ,$$

so that

$$a = \frac{f}{m} = 5.5 \times 10^{-5} \frac{A_c}{m} \text{ cm sec}^{-2} \quad .$$

For the proposed satellite,

$$A_c = 1521 \text{ cm}^2 \quad ,$$

and

$$m = 6.8 \times 10^5 \text{ g} \quad ,$$

so the force exerted by direct solar photon pressure is

$$f = 0.084 \text{ dyne}$$

and

$$a = 1.2 \times 10^{-7} \text{ cm sec}^{-2} \quad .$$

Direct solar photon pressure will produce orbital perturbations with a period equal to that of the orbit. The amplitude of the fundamental frequency component of this perturbation is greatest when the sun line is in the orbit plane and has been calculated, by using the acceleration noted above, to be about 10 cm peak to peak for LAGEOS. The solar constant is known to 1% and is believed to be constant to within 0.6%, so the favorable geometry and surface characteristics of LAGEOS should, conservatively, allow this perturbation to be calculated to much better than 10%. Therefore, the uncertainty should not exceed 1 cm.

Other periodic perturbations are produced by direct sunlight – e.g., the harmonics of the (fundamental) orbit frequency – but their amplitudes are small. There are also long-period (e.g., perigee period and nodal period with respect to the sun) as well as secular components of the solar photon perturbation. However, solar photon pressure generally tends to cancel in a near-circular orbit when averaged over a full revolution, so the magnitudes of these effects for a satellite with a high mass-to-area ratio are not excessive. These perturbations have not yet been fully analyzed, but rough estimates indicate that for a 24-hr arc, the magnitudes do not exceed 10 cm for LAGEOS. This can also be computed to better than 10%.

Unlike direct solar radiation, earthshine is variable in magnitude and roughly constant in direction relative to the satellite velocity vector. Thus, the effects on the orbit do not balance out but tend to be cumulative. The average planetary albedo has been variously estimated to be 30 to 40% (cloud cover, about 50% or higher; and ground and atmosphere, about 20%). The combined albedo and thermal radiation from the earth will thus exert a force that will vary in both intensity and direction as a function of time and geography.

The earth is very nearly in overall thermal equilibrium, so the total energy reflected or reradiated is equal to the incident sunlight. Thus, the average energy flux density through a spherical surface concentric with the earth and at the LAGEOS altitude is 10% of the direct solar-energy density at the earth. However, earthshine will exhibit wide geographic and day-night variations, which can be as large as 20% of direct sunlight.

We estimate that earthsine can produce orbital perturbations as large as 40 cm or so in a 24-hr arc of LAGEOS. We should like to reduce the orbital error caused by earthshine to less than 10% of this value. A direct approach would be to increase the m/A_c of the satellite, but a substantial increase would be quite difficult to attain because a much larger launch vehicle would be required. Another means of reducing this error would be to increase the orbital altitude. However, a rather large altitude increase would be needed to effect a significant reduction, and again, a larger launch vehicle would be necessary to maintain a constant m/A_c. The best recource is to model this force as carefully as possible, obtain cloud-cover data from meteorological satellite observations, and then calculate these forces after the fact. It is not clear just how accurate such calculations will be, but a 10% correction seems feasible and will result in acceptable orbit accuracies if the satellite m/A_c is large enough.

The required mass-to-area ratio can be estimated in the following way. We set the required orbit accuracy at 5 cm for a 24-hr arc and note that the orbit error

$$\delta s = \frac{1}{2}\, \delta a t^2 = 3.7 \times 10^9\, \delta a \quad .$$

If the earthshine variations are ± 10% of the direct solar photon flux, and if earthshine can be modeled to an accuracy of ± 10%, then

$$\delta a = 1\% \text{ of direct solar} = 5.5 \times 10^{-7} \times \frac{1}{m/A_c} \text{ cm sec}^{-2} \quad ;$$

therefore,

$$\delta s = 5 \text{ cm} = 3.7 \times 10^{9} \times 5.5 \times 10^{-7} \times \frac{1}{m/A_c} \quad ,$$

$$\frac{m}{A_c} = 4 \times 10^{2} \text{ g cm}^{-2} = 4 \times 10^{3} \text{ kg m}^{-2} \quad ,$$

and we establish a requirement that

$$\frac{m}{A_c} \geqq 4000 \text{ kg m}^{-2} \quad .$$

Station-position errors resulting from earthshine orbit perturbations should not exhibit a completely systematic bias in any one direction for all satellite passes. With a set of data comprised of a balanced mixture of day and night passes, northbound and southbound passes, and passes to the west and to the east of the station, all with varying pass elevation angles, a 5-cm (earthshine) orbital error will result in an error in computed station position of less than 2 cm.

LAGEOS will absorb some 60 to 70 w of incident solar radiation and earthshine, depending on how much of the orbit is in the earth's shadow. The reradiation of this absorbed energy will also exert a force on the satellite. The net force acting to perturb the satellite trajectory will depend on the asymmetry of the reradiated flux, and the latter in turn will be determined by the variations in temperature and emissivity over the satellite surface.

Under equilibrium conditions when illuminated by the sun, the satellite core will have an average temperature of about 265 K, with a gradient across the satellite of 3°C. The retroreflectors facing the sun will be at 277 K; those facing space, 249 K; and those facing the earth, 261 K. The retroreflectors, with an emissivity of 0.85, will reradiate some two-thirds of the absorbed energy; and the diffuse aluminum surface, with an emissivity of 0.34, will reradiate about one-third. The unbalance in the flux radiated by the cube corners will be about 8 w, and that for the aluminum surface area, much less than 1 w. These two unbalanced photon fluxes will result in a net force on the satellite of roughly 0.003 dyne, or about 4% of the force of direct solar radiation. It should be noted that this force is not strongly influenced by orbital altitude and that the satellite acceleration produced (5×10^{-9} cm sec^{-2} for LAGEOS) can be controlled only by maintaining a large mass-to-area ratio.

Uncertainties in the force exerted on the satellite by photon pressure will be the limiting factor in the accuracy of computed LAGEOS orbits. The only positive means of reducing this error are to make certain that the satellite's surface reflectivity and geometry are symmetric, known, and stable and to increase the satellite's mass-to-area ratio to the highest possible extent.

The force exerted by atmospheric drag can be calculated from

$$f_d = \frac{C_d}{2} A_c \rho v^2 \quad .$$

We will assume $C_d = 2$ and v = satellite orbital velocity = 6.3 km sec^{-1}. Jacchia (1970) tabulated values of atmospheric density ρ for altitudes up to 2000 km. We have plotted his values of ρ and extrapolated "by eye" up to an altitude of 3700 km for exospheric temperatures of 850 and 2000 K. These temperatures correspond to minimum and maximum air densities, respectively. We obtained the results shown in Table 3. Although our estimates of air density at this altitude are somewhat uncertain, it appears that atmospheric drag will not have any significant influence on orbit accuracy.

A rough estimate based on the extrapolated atmospheric densities shown in Table 3 yields an orbital lifetime for the proposed satellite of more than 10 million years.

Table 3. Estimated magnitudes of nongravitational perturbing forces and resulting accelerations for a spherical satellite with a mass-to-area ratio of 4000 kg m^{-2} in a circular orbit of 3700-km altitude.

Source of perturbation		Acceleration ($\times 10^{-9}$ cm sec^{-2})
Direct solar photon pressure	0.084	120
Earth albedo and thermal radiation	variable up to 0.02	up to 30
Unbalanced satellite reradiation	0.01	12
Atmospheric drag	1×10^{-5} to 4×10^{-4}	0.01 to 0.4
Micrometeorite impact	2.4×10^{-6}	0.004

Using the data presented by Whipple (1968) for the cumulative impact rates of meteoritic material on a sphere near the earth's orbit and integrating the curve shown in Figure 1 of that paper, we have obtained the total mass of such material of all sizes that will strike our satellite:

$$2 \times 10^{-16} \text{ g cm}^{-2} \text{ sec}^{-1} .$$

The total surface area of a sphere of 22-cm radius is 6080 cm^2, so the rate of impacting meteoritic material will be

$$1.2 \times 10^{-12} \text{ g sec}^{-1} .$$

If we assume the average velocity of meteoritic particles is 20 km sec^{-1} and that all particles that impact on the satellite are absorbed, then the total momentum of the particles will be transferred to the satellite. Thus,

$$\frac{d}{dt}(mv) = 2.4 \times 10^{-6} \text{ g cm sec}^{-2} .$$

For the satellite mass of 680, 000 g,

$$a = 3.6 \times 10^{-12} \text{ cm sec}^{-2} .$$

The estimate of meteoritic impact rates should be reliable to within a factor of 2; furthermore, meteoritic particles will actually strike the spacecraft from all directions, so that momentum contributions will tend to cancel, reducing the above estimate. It appears that meteoritic impact will not have a significant effect on the satellite orbit. There is, of course, the remote possibility that a large particle will strike the satellite. The probabilities for such events are quite low, being less than 1 impact per 3000 yr for 1-mg micrometeorites and 1 impact per 10^8 yr for particles with a mass of 1 g.

5. SATELLITE DESIGN

5.1 Size and Shape

As noted earlier in this report, LAGEOS must be spherical in order to provide the isotropy required for accurate modeling of photon pressure and for accurate laser ranging.

The dimensions of the satellite must be adjusted to satisfy several competing criteria:

A. Minimum size to attain the greatest possible mass-to-area ratio compatible with the given launch-vehicle payload-weight capacity.

B. Minimum size to reduce errors in range observations that result from finite satellite geometry.

C. Large enough size that the resulting satellite density can be realized with actual and available materials.

D. Large enough size to accommodate the required array of retroreflectors.

E. Large enough size to be detected by cameras with a large field of view for initial or subsequent orbit acquistion.

A careful analysis of these criteria has been completed under this study grant; we conclude that criteria D and E are satisfied for a spherical satellite having the maximum density that can be practically realized, viz., one that satisfies criterion C. This configuration obviously results in dimensions that provide the best attainable response to criteria A and B.

These analyses are discussed in detail in the remainder of this report.

5.2 Camera Visibility

The effective field of view of present laser systems is quite restricted, typically to a maximum of 5 or 10 mrad. In some systems, an auxiliary telescope is provided as a visual acquisition aid to circumvent the aiming problem that results from the narrow field of view of the laser. However, LAGEOS will be too faint an object for these telescopes if its dimensions are kept small enough to satisfy the other requirements.

Thus, we propose that initial orbit acquisition be accomplished with the existing network of Baker-Nunn cameras, which were expressly designed for such tasks. The along-track field of view of these cameras is 30° (~ 500 mrad), and their sensitivity is very high because they "track" at satellite rates and can thus use long exposure times to build up a faint satellite image. This approach has the advantage that no instrumentation of any kind is needed on the satellite. It is only necessary that we provide a good reflective surface on the approximately 50% of the sphere that cannot, in any event, be covered with laser retroreflectors.

With a diffusely reflecting aluminum coating over 58.7% of its surface, we calculate an apparent magnitude (including atmospheric absorption) for the reflected sunlight from LAGEOS to be 12.96 at an elevation angle of 45°. This should produce marginally usable Baker-Nunn images for an exposure time of 0.8 sec. Since LAGEOS will have a rather slow angular rate as seen from the camera, 350 arcsec sec^{-1}, much longer exposure times can be used – e.g., 3.2 sec – to produce good images.

To verify our calculations, we tested the Baker-Nunn sensitivity by photographing Vanguards 1 and 2. Good images were obtained for the latter with exposure times of 0.8 sec and less. Comparing one observation at a range of 3.40 Mm for the 26-cm-radius Vanguard 2 with a similar LAGEOS observation at a range of 4.5 Mm, we find that equivalent images would be obtained for LAGEOS with an exposure time of 1.88 sec. The fainter Vanguard 1 (with a radius of 8 cm) was successfully photographed at exposure times up to 30 sec. Since the apparent angular rates of LAGEOS and Vanguard 1 are comparable, the LAGEOS exposure times can also be of a similar duration. We conclude that the 22-cm radius of LAGEOS is sufficiently large for routine observation of the satellite with Baker-Nunn cameras.

5.3 Retroreflector Array

5.3.1 Configuration

The requirements for the retroreflector array are that it must reflect enough energy into a telescope collocated with a laser transmitter to allow accurate range measurements, that these range measurements can be accurately related to the LAGEOS center of mass, that both amplitude and range accuracy requirements should be satisfied for all LAGEOS aspect or viewing angles, and that these properties should be stable for an extended life in orbit.

Perhaps the first question to be answered in designing the array is whether we should use a small number of cube corners arranged so that only one element at a time produces retroreflection, thus eliminating complications that result from having some larger number of coherent reflections. However, unless the cube corners are hollow and the vertex of each cube-corner retroreflector is placed at the LAGEOS center of mass, the difference in range between the retroreflector and the center of mass will be a function of aspect angle. For a satellite with the dimensions of LAGEOS, this range difference will produce a systematic range error that exceeds our accuracy requirement and cannot be corrected unless some means is provided for determining unambiguously the satellite aspect angle. Furthermore, if this range error is eliminated by placing the vertex of each hollow cube corner at the satellite center of mass, too much of the spacecraft volume will be occupied with cube corners, and we cannot in any practical way meet the required mass-to-area ratio or (cross-section) isotropy for surface-force modeling.

We conclude that the LAGEOS retroreflector array must consist of a "large" number of reflecting elements, where "large" is the minimum number needed to control variations in observed range with aspect angle. These variations have been calculated for arrays of different numbers of cube corners distributed uniformly over the LAGEOS spherical surface. The result has been the selection of an array of 240 cube corners; this number yields a maximum variation in observed range as a function of aspect angle of 1.2 mm and yet is from the standpoint of cost and manufacturing complexity.

5.3.2 Cube-corner design

Retroreflector Type. A basic decision for LAGEOS was between an open or a solid retroreflector. We have chosen the latter for the following reasons:

A. The state of the art in the fabrication of solid retroreflectors is well advanced – certainly beyond the requirements for LAGEOS. On the other hand, manufacturing techniques of open retroreflectors of the quality and precision required have not yet been established. There is some evidence, however, of progress toward this goal, by means of replication by electroforming processes, which seems to be the only practical means of making hollow cube corners at this time.

B. There is a serious question as to the long-term mechanical stability of electroformed cube corners, and until experience shows otherwise, we must assume that they may not be able to maintain their figure to optical tolerances over a period of a decade or more, particularly when built up to the larger thicknesses needed to meet mechanical and thermal requirements. In contrast, years of experience have proved that optical elements made from high-grade fused silica are notable for their long-term stability, and we can be completely confident that cube corners made from this material will retain their optical properties over many years in orbit.

C. The primary advantage of a hollow cube corner – the absence of any problem with thermal gradients distorting the retroreflector beam patterns – is not of critical importance for LAGEOS. The thermal analysis described in Appendix A and analysis of the array transfer function together indicate that thermal gradients in a properly designed solid cube corner will not cause significant range errors in the case of LAGEOS.

Cube-Corner Aperture Shape. Early LAGEOS concepts utilized triangular-aperture, solid retroreflectors because of their retroreflection characteristics at high-incidence angles are better than those of hexagonal or circular aperture. However, this advantage is now considered to be more than offset by the problems associated with the actual mounting of these triangular retroreflectors – such as aperture shadowing, mechanical distortion, and difficult thermal control – as well as by the greater difficulty in fabrication and hence higher cost per aperture area. Consequently, the triangular aperture has been discarded.

Of the other two possibilities, hexagonal and circular, we have selected the latter as best for LAGEOS. Although some advantage may be gained in a larger total aperture area if the hexagonal shape is used, this consideration is offset by the superior mounting that can be employed with a circular front face. The cube-corner-mounting problem was examined in detail by A. D. Little, Inc., during the design of the Apollo lunar retroreflector array (A. D. Little, 1969). They concluded that a mounting using a threaded retainer was best. We agree with this evaluation and have therefore decided on circular front-face cube corners. Indeed, the LAGEOS cube corner and mounting now proposed are essentially the same as those for the lunar retroreflector arrays, so a significant amount of experience, analysis, and design data can be utilized in the LAGEOS program.

Retroreflector Size. To provide adequate clearance for mounting, a practical maximum of about one-half the spherical surface can be devoted to an active retroreflector area. Within this constraint, the size of the individual retroreflectors is determined by the total number of them (240) and the reflecting area required. Accordingly, the diameter of the aperture has been set at 3.65 cm (1.438 inches).

Retroreflector Material. The material to be used for the retroreflectors must be optically homogeneous, have good optical transmissibility in the visible spectrum, and possess high long-term mechanical and thermal stability. Further important requirements are properties such as low coefficient of thermal expansion; adequate workability; high resistance to moisture and to chemical, radiational, and thermal degradation; low coefficient of refractive-index change with temperature; and ready availability at reasonable cost. Candidate materials include traditional optical glasses, the new tailored optical materials, and natural materials like quartz and sapphire; of these, one material – synthetic, fused silica – best meets all the requirements. This material, in fact, has been used for all the laser retroreflectors now in orbit or on the moon.

A number of grades of fused silica are available from the several manufacturers of such optical material. The grades differ in chemical purity and optical clarity, both important factors. Low chemical purity causes changes in color and transparency on exposure to ultraviolet and other radiation, so we must select the purist fused silica reasonably available in order to minimize long-term degradation due to radiation exposure.

All optical materials have inherent internal inhomogeneities and minute inclusions such as straiae, bubbles, and seeds. These result in a distortion of the wavefront of the incident laser beam as it progresses through the retroreflector.

The LAGEOS specification calls for the same type of fused silica that was used for the Apollo retroreflector arrays and that is to be used for GEOS C. This selection is judged to be acceptable in terms of cost, availability, and optical performance.

Retroreflector Geometry. A theoretically perfect retroreflector has three reflective surfaces and an entrance face, all perfectly flat; the dihedral angles between each pair of reflective faces are exact; and the angles each reflective face makes with the entrance face are all equal. The actual retroreflectors depart from perfect geometry: The return beam may deviate in direction, or it may have greater divergence from or differ in energy distribution or symmetry; more likely, the beam will contain some combination of all these effects. Past experience with precision retroreflectors, particularly those used in the lunar retroreflector arrays and the GEOS satellites, has shown that present fabrication techniques can provide, at reasonable costs, retroreflectors with geometric accuracy sufficiently close to the perfect case that no significant performance degradation occurs.

In order to provide measurable echo signal strengths, the retroreflectors must reflect the incident pulses into an extremely narrow beam along the line of sight back to the laser station. If the retroreflector beamwidth is too narrow, however, the angular displacement (of the retroreflected beam) caused by velocity aberration will prevent the echo from entering the receiving telescope. The velocity aberration for the LAGEOS orbit will range from 26 to 42 μrad for various satellite–ground-station geometries.

The LAGEOS retroreflectors will have a circular front-face aperture 3.65 cm in diameter. A dihedral angle offset by 1.75 ± 0.5 arcsec will produce the desired beamwidth. We have computed the retroreflected beam pattern for fused-quartz cube corners of this configuration. The result is shown in Figure 1.

Diffraction will cause the beam pattern to broaden as the angle of incidence departs from the normal to the front face of the retroreflector, thereby reducing the

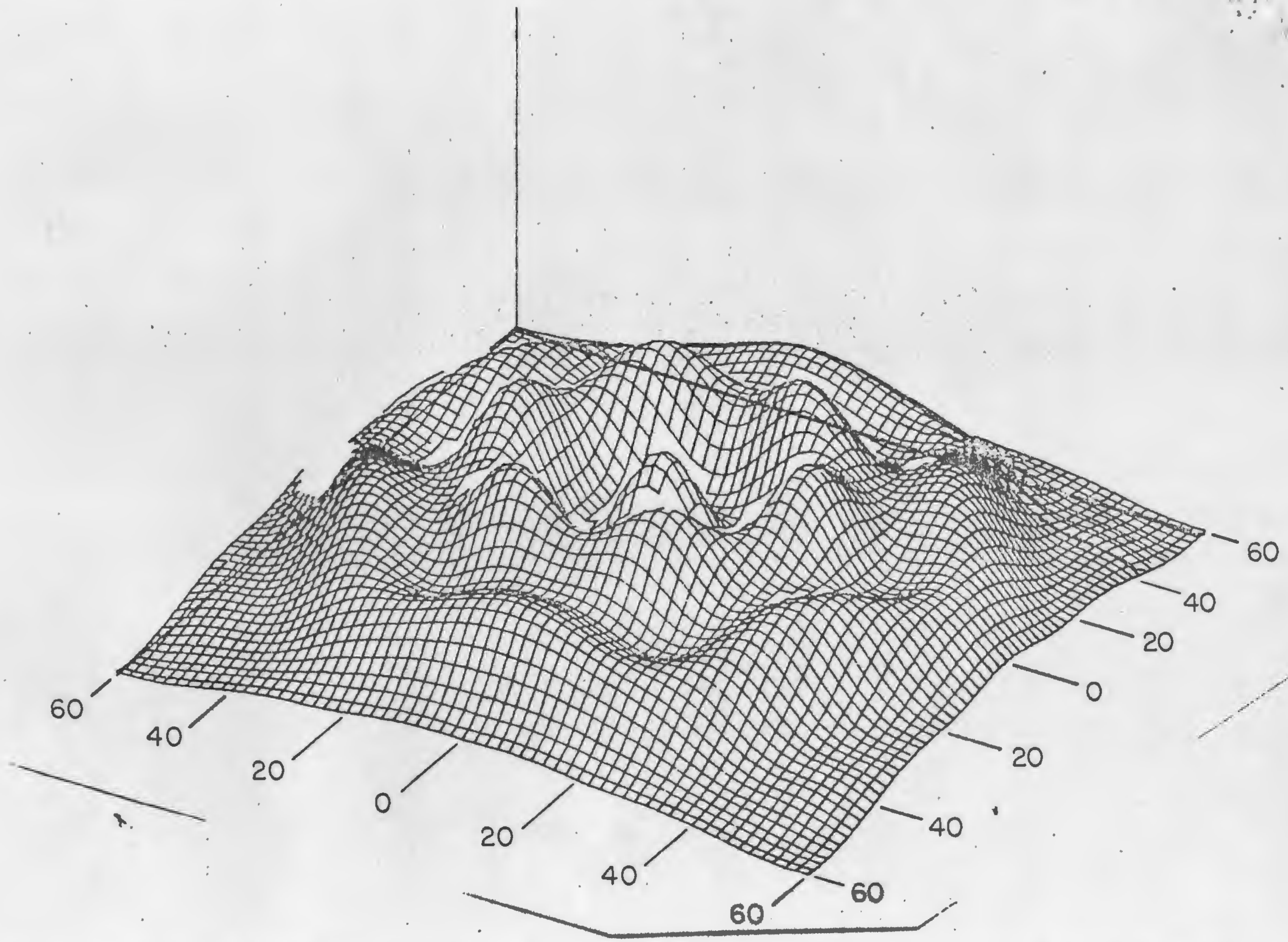

Figure 1. Beam pattern at normal incidence for an uncoated cube corner with a circular aperture 3.65 cm in diameter. Dihedral angle is 8.5 μrad (1.57 arcsec) from 90°. Beam angles are given in microradians. Wavelength is 6943 Å (ruby laser).

reflectivity. This effect is in addition to the (geometric) reduction of the effective aperture area with increasing incidence angle. Figure 2 plots the effective reflectivity of a LAGEOS cube corner as a function of incidence angle.

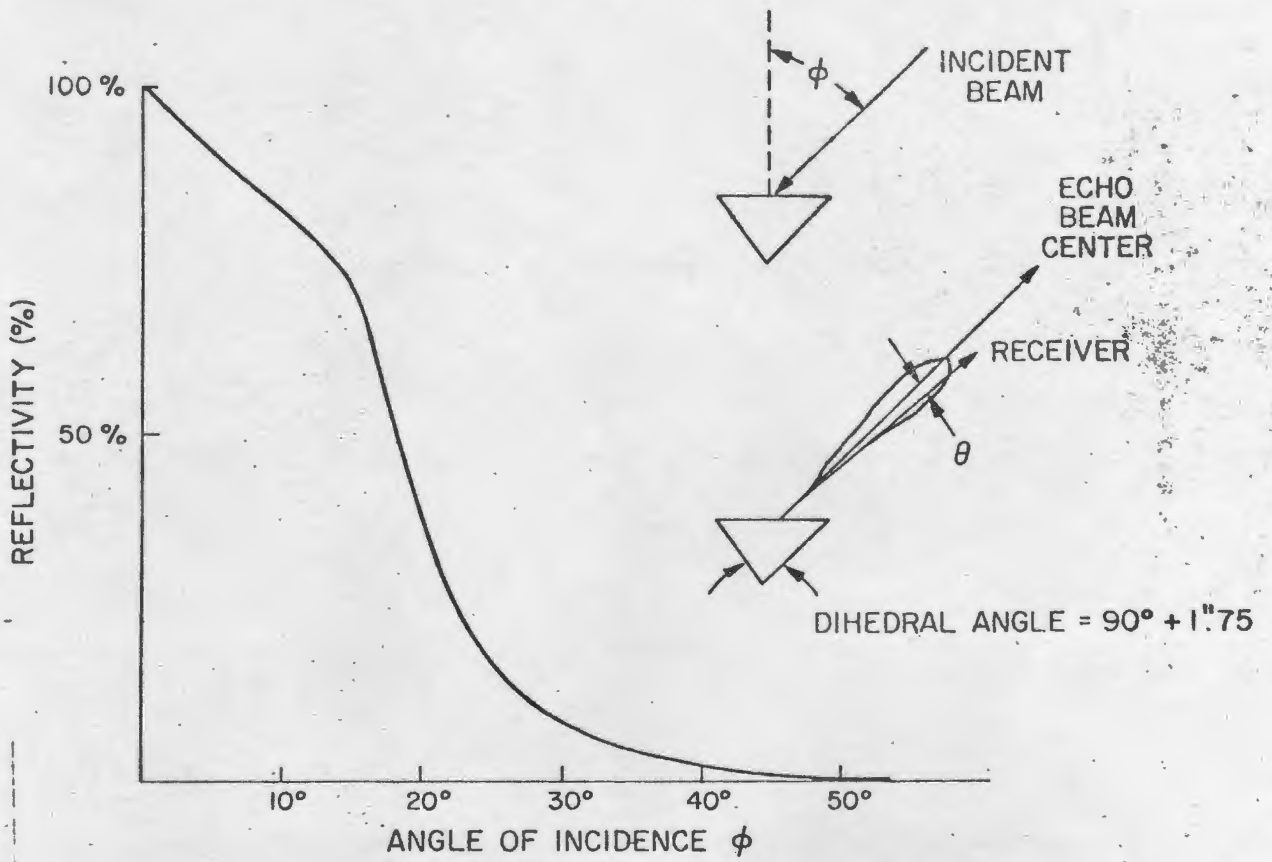

Figure 2. Reflectivity versus angle of incidence for an uncoated fused-silica cube corner with a circular aperture 3.65 cm in diameter and dihedral angles of 90° + 1.75 arcsec. The reflectivities are for a beam angle θ of 36-μrad, corresponding to a typical value of velocity aberration for the LAGEOS orbit. The reflectivity is the average for all azimuthal angles (taken around the normal to the front face).

As discussed above, the reflected beam must be tailored to match the velocity aberration associated with the actual orbit of the satellite. Accordingly, the energy distribution in the reflected beam must be optimized within a symmetrical conical zone coaxial with the incident beam. The half-angles of this conical zone for the predicted satellite orbit are 30 to 40 μrad, and it is desirable that the distribution be most dense at the greater angle.

The best method to tailor the reflected beam to this requirement is to make the dihedral angles for all three reflecting surfaces 90° + (1.75 ± 0.5) arcsec.

A retroreflector tailored to produce a specific energy-distribution pattern in the reflected beam as described above must still adhere closely to the perfect retroreflector geometry in all other respects; i.e., the reflective and entrance faces must all be flat, and the angles between each reflective face and the entrance face must be equal. Deviations from these geometric ideals will alter the energy-distribution pattern of the reflected beam. Consequently, the cube-corner specification calls for dihedral angles accurate to within 0.50 arcsec and surface flatness to within λ/10 (both tolerances are identical to those for GEOS C).

Retroreflector Coating. Two distinct and separate decisions on coating must be made for the LAGEOS retroreflectors:

A. Should the reflective faces be coated, and if so, what type of coating should be applied?

B. Should the entrance face be coated?

Relative to the first, a solid retroreflector with no coating reflection. However, retroreflection is restricted to cases where the incidence angle at any one face of the ray as it passes through the retroreflector is within the internal reflection angle for that material. This limits the incidence angle of the incident beam for which retroreflection will occur; for fused silica, this angle ranges from 16°9 to 57°4, depending on the azimuth angle of the incident beam.

If the faces have a reflective coating (usually metallic), the ray, as it passes through the retroreflector, will be reflected at each face regardless of its incidence angle. Accordingly, retroreflection is obtained over a larger angle of the incident beam. This angle is 57°4 for a fused-silica retroreflector with a circular entrance face or aperture and is independent of the azimuth angle of the incident beam. Thus, a reflective coating enhances retroreflection at larger incidence angles, an important factor for planar arrays that can be viewed at incidence angles far from normal incidence.

However, a spherical array will always be viewed with some number of cube corners at near normal incidence. In addition, total internal reflection – when it does take place – is quite literally almost total, while reflection from a metallic coating suffers some loss. For aluminum coatings, the reflectance at the ruby wavelength (6943 Å) is 0.897; and since every retroreflection requires a light beam to be reflected from all three cube-corner reflecting surfaces, the effective reflectance is $(0.897)^3$, or 0.722. We have calculated the actual return-pulse amplitudes for the LAGEOS array for both cases, coated and uncoated; the results are reflectances equivalent to 12.89 and 7.42 cube corners at normal incidence, without losses. With a reflectance of 0.722 for the coated case, the numbers are 9.30 and 7.42 – or echo strengths in the ratio of 5:4 for the coated and uncoated cases, respectively.

This difference in echo strengths is not critical, because the LAGEOS return-pulse amplitudes for contemporary laser systems are sufficient in either case. Therefore, the LAGEOS specifications call for uncoated retroreflectors. This simplifies the fabrication of the cube corners and completely eliminates any question as to the adherence of the reflective coatings after some years in orbit. The latter concern is not significant in terms of loss of echo strength, but it is very significant in terms of range accuracy, since any degradation in reflectance that is not uniform over the array can effectively change the array transfer function so that it varies systematically with aspect angle. This, in turn, could introduce significant systematic errors into the range observations.

The second area for coating considerations is the retroreflector entance face. An antireflection coating on this face could increase the overall intensity of the reflected beam by reducing the loss due to reflection as the incident beam strikes the entrance face. This increase in efficiency is small, however. Such coatings are wavelength-sensitive and cannot be designed to be effective for all possible future laser wavelengths. Furthermore, the wavelength at which these coatings are effective is a function of the angle of incidence, which is obviously incompatible with the fact that LAGEOS has no constant and unique incidence angle for the incident pulses.

Since there is a possibility that direct exposure to solar and other radiation might in time seriously degrade such a coating and hence reduce its transmissibility, the probability is high that the small gain in efficiency will be more than offset by a potentially larger loss in efficiency, and, as in the case of the reflective coatings, could introduce systematic range errors.

5.3.3 Transfer function

With such an array, the echo signal arriving at the photoreceiver will be the vector sum of the mutually coherent retroreflected pulses from a number of individual retroreflectors. In order to calculate the range to the satellite center of mass, we must fashion a transfer function that relates the time of arrival of this observed echo to the center-of-mass range. This transfer function must be synthesized from the geometry and the reflecting characteristics of the individual reflectors. It will be shown that the transfer function for LAGEOS is a constant range increment that is simply added to each observation.

The transfer function must take into account the geometric location of each cube corner relative to the satellite center of mass, the incidence angle of the laser beam for each cube corner, the beam pattern of each cube corner, and the relative phases of the individual return pulses from the cube corners. It is essential that the transfer function not vary significantly with satellite aspect angle, and we must configure the retroreflector array to ensure that this condition is met to sufficient accuracy.

At satellite ranges and over the small area of a retroreflector array, laser pulses incident on a satellite have essentially plane wavefronts; i.e., the incident light is coherent over the dimensions of the array. This fact must be considered when the characteristics of the echo pulse are calculated, because the resulting interference effects produce significant changes in shape and large changes in amplitude of the received pulses. We have performed detailed calculations of the retroreflected pulses from LAGEOS, and the results are presented below. First, we give a rather simplified description of the reflection process, in order to elucidate the characteristics of the LAGEOS array.

We assume the laser light is monochromatic and omit all factors common to the reflected signal from all the cube corners, such as range attentuation and pulse energy. These conditions will not limit the generality of our conclusions, but will simplify the discussion.

Under these assumptions we can let the pulse from the laser be represented by the vector

$$\vec{u} = g\left(1 - \frac{x}{c}\right) \exp i\,\omega\left(t - \frac{x}{c}\right) \quad , \tag{1}$$

where x is the direction of propagation, the function $g[t - (x/c)]$ represents the envelope of the pulse, and $\exp i\omega[t - (x/c)]$ represents the (optical) oscillations within the pulse. If the distance from the laser to the satellite center of mass (CM) and back to the photoreceiver is S, then the range to the i^{th} cube corner and back will be $S - 2d_i$ (see Figure 3). If two retroreflectors i and j return echoes to the receiver, the received signal will be the vector sum of

$$u_i = a_i g\left[t - \frac{1}{c}(S - 2d_i)\right] \exp i\omega\left[t - \frac{1}{c}(S - 2d_i)\right] \tag{2}$$

and

$$u_j = a_j g\left[t - \frac{1}{c}(S - 2d_j)\right] \exp i\omega\left[t - \frac{1}{c}(S - 2d_j)\right] \quad , \tag{3}$$

where a_i is a coefficient dependent on the effective reflectivity of the i^{th} cube corner. The intensity of the resultant will be

$$I = \left|u_i + u_j\right|^2 \tag{4}$$

$$= a_i^2\, g_i^2 + a_j^2\, g_j^2 + 2a_i a_j g_i g_j \cos\theta \quad , \tag{5}$$

where the phase angle θ is

$$\theta = -\frac{\omega}{c}\left[(S - 2d_i) - (S - 2d_j)\right] \tag{6}$$

$$= 4\pi\left(\frac{d_i - d_j}{\lambda}\right) \quad . \tag{7}$$

The first two terms on the right side of equation (5) for I represent the resultant of two incoherent pulses, and the third is the interference between two coherent pulses.

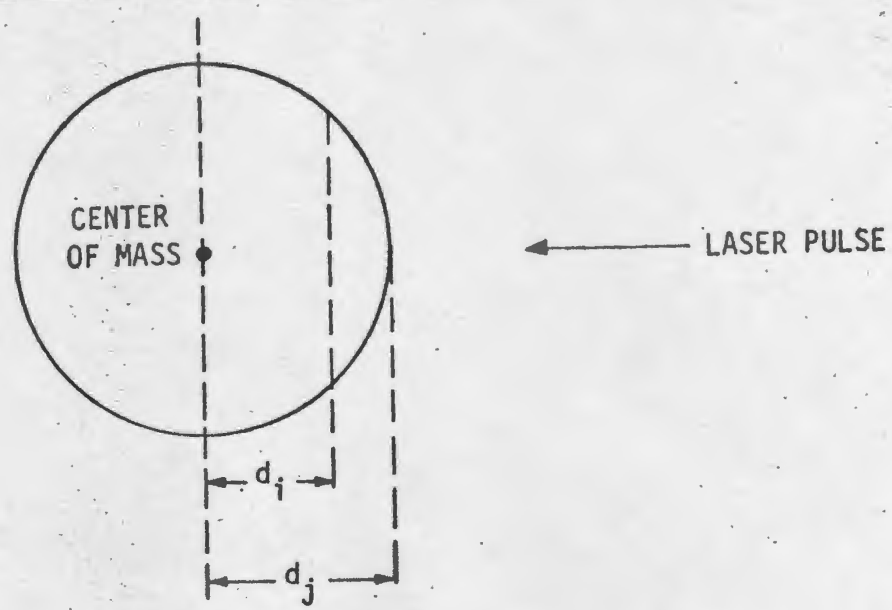

Figure 3.

Figure 4 shows the time history of the echo signal as it arrives at the receiver. The pulse at time t_{CM} is the pulse that would be received from a point reflector at the satellite center of mass, and the pulses at t_i and t_j are those from the i^{th} and j^{th} cube corners. It is t_{CM} we wish to know since we need it to calculate the range to the satellite center of mass. However, it is the composite resultant of the pulses at t_i and t_j that we can observe. Thus, we must determine the relationship between the observed times of arrival of the composite pulses and the corresponding values of t_{CM}. We must then examine the influence of the satellite configuration on this relationship, or transfer function, and then adopt a configuration that results in a transfer function of acceptable accuracy and stability.

The intensity of the composite pulse will be $a_j^2\ g_j^2$ in the time interval AB and $a_i^2\ g_i^2$ in the interval CD; but in the overlap (interference) region BC, the intensity must be calculated by using all terms in the equation for I above.

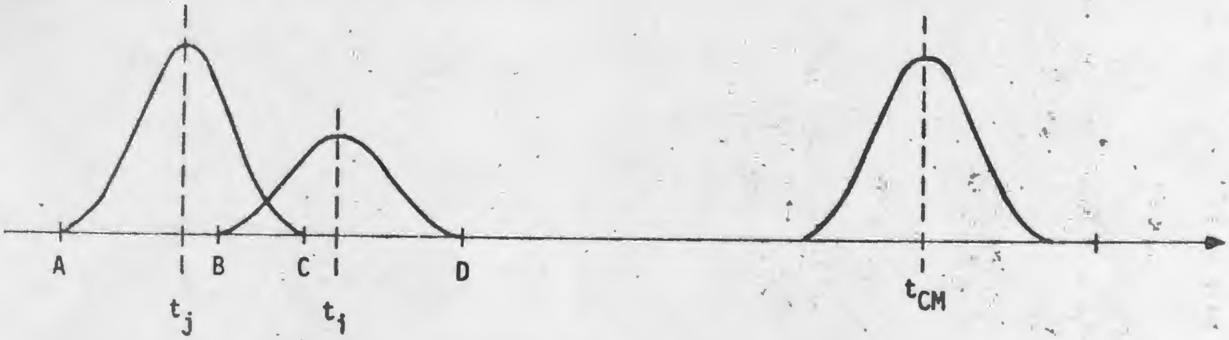

Figure 4. Time history of the echo signal.

It is apparent that the phase angle θ is very sensitive to the difference in relative range to the two cube corners, since the intensity in the overlap region of the received pulse will change from a minimum (destructive interference) to a maximum (constructive interference) for a change in $d_i - d_j$ of only a quarter-wavelength of the laser light. (For a ruby laser, $\lambda/4 \sim 2 \times 10^{-4}$ mm.) As a practical matter, we must therefore consider the phase angle θ to be indeterminate and must expect that the shape, amplitude, and centroid will fluctuate from pulse to pulse as a result of this interference.

The relationship of the centroid of the received composite pulse to t_{CM} is also a function of d_i and d_j through their direct influence on t_i and t_j, although this effect is much less sensitive to changes in d_i and d_j than is the phase. The cube-corner reflectivities influence the composite pulse shape through the coefficients a_i, and thus also affect the relationship between the pulse centroid and the satellite center of mass.

In order to avoid any requirement for knowing the distances d_i for every laser observation, and therefore the LAGEOS orientation, the satellite will be uniformly covered with a sufficient number of cube corners that the observed time of arrival of the received-pulse centroid, when averaged over some reasonable number of measurements, can be related to t_{CM} with an accuracy of 3 mm for any combination of aspect angles.

A reflected pulse will have contributions from all the cube corners on the shaded spherical segment shown in Figure 5, where ϕ_c is the cutoff angle for the specific cube-corner design used. For an uncoated solid fused-silica cube corner with a circular aperture, $\phi_c = 57°45$, so the active cube corners will be distributed in range (the total variation of the d_i) over h = 10.2 cm for the 22-cm radius of LAGEOS. Using the specified LAGEOS cube-corner characteristics, we have calculated the magnitudes of the contributions (omitting interference effects) to the reflected signal from spherical segments with $\Delta h = 0.5$ cm. The result is shown in Figure 6. Some 55 of the 240 cube corners are "active," and the effective reflectivity is equivalent to 7.4 cube corners at normal incidence.

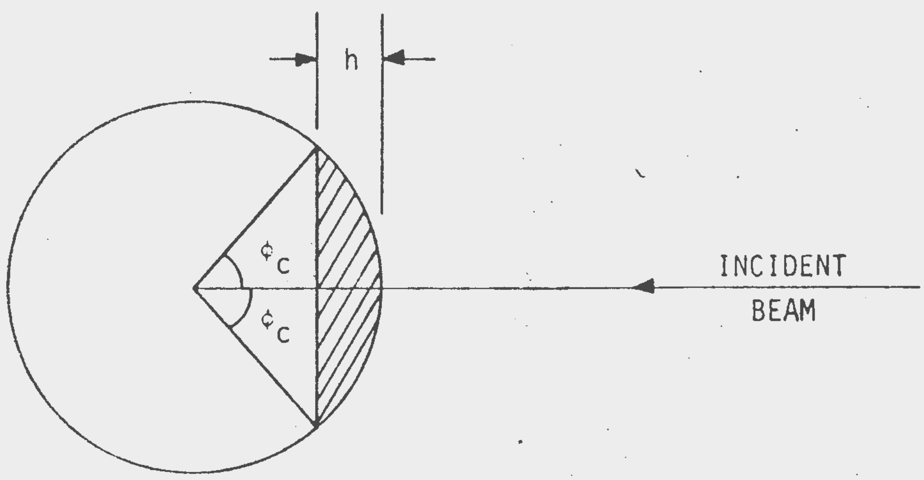

Figure 5. The shaded area is the spherical segment where the "active" cube corners are located when ϕ_c is the cube-corner cutoff angle.

If we extend the equations for two retroreflectors to the case of n retroreflectors, the intensity of the reflected pulse will be the square of the vector sum of the individual echoes from each active retroreflector,

$$I = \left| \sum_i u_i \right|^2 \tag{8}$$

$$= \sum_i a_i^2 g_i^2 + \sum_{i,j} a_i g_i a_j g_j \cos \theta_{ij} \quad , \tag{9}$$

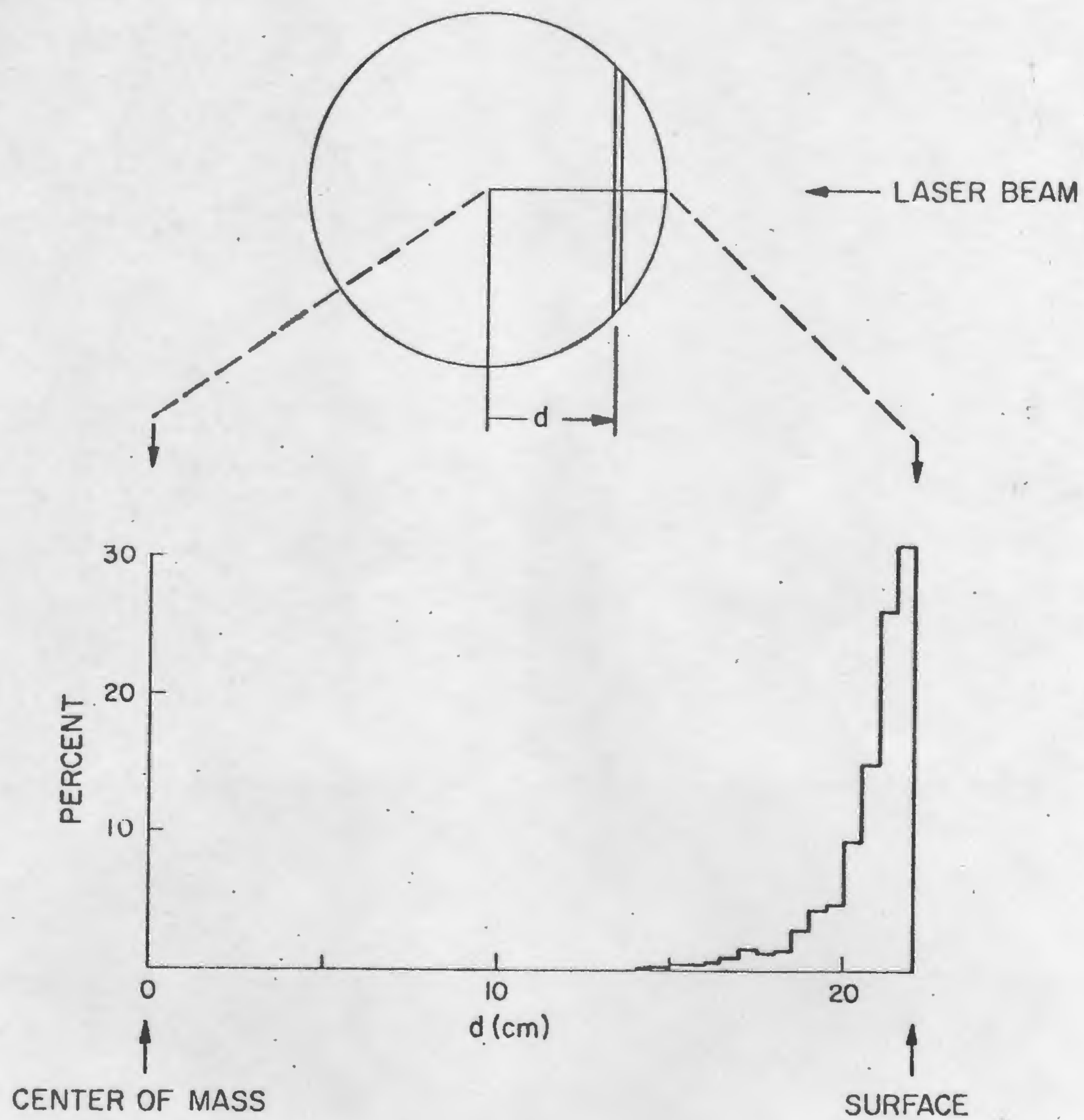

Figure 6. Effective reflectivity of segments of the LAGEOS retroreflector array versus the distance d from the center of mass. The histogram is the average over results computed for several aspect angles and for incoherent light; i. e., interference effects have been omitted. It is of interest to note that 57% of the total return is from the first centimeter, and 90% from the first 3 cm of the sphere.

where the phase angle θ_{ij} is

$$\theta_{ij} = 4\pi\left(\frac{d_i - d_j}{\lambda}\right) . \tag{10}$$

This equation can be used to compute the intensity as a function of time – i.e., the pulse shape – for the signal received from a specified retroreflector array once the geometry of the array and the reflecting characteristics of the retroreflectors are determined. It is then possible to compute the relationship between the time of reception of a pulse centroid and the range to the satellite center of mass. Since there will be no feasible way to determine the phase angle θ_{ij} when LAGEOS is in orbit, this relationship must be treated statistically. Further, since the orientation of LAGEOS will not be known, variations with satellite aspect of the pulse-centroid/center-of-mass relationship must also be described in statistical terms. This circumstance is quite acceptable if the statistical variations have a stable and well-defined mean and if the variations converge to this mean value to within our accuracy requirements for a reasonably small number of observations (e.g., the number of returns that can be acquired in one pass). Another question of importance is how large are the fluctuations in received signal strength that are caused by interference effects.

The terms in equation (9) containing cos θ_{ij} change from maximum to minimum for an aspect-angle change of the order of 0.5 arcsec. The sum in equation (9) over terms lacking a cos θ_{ij} factor is, by design, insensitive to changes in aspect. Consequently, because the terms in cos θ_{ij} must converge to zero for a sufficiently large number of range measurements, the mean centroid location relative to the satellite center of mass must be that calculated for the incoherent terms in equation (9), i.e., the first summation. For LAGEOS, this range correction is 15.8 cm.

To verify the expected convergence of the range correction to that for the incoherent case, we have computed the amplitudes and centroid positions for a number of retroreflected pulses from LAGEOS, using incident laser pulses with gaussian shapes and different pulse widths. Since all values of phase angle (from 0 to 2π) are equally probable, values of θ_{ij} were obtained from a random-number generator. The aspect angle was also allowed to vary. The results are given in Table 4. It is apparent that the expected results are obtained; therefore, the transfer function for LAGEOS is very simple – viz., the constant range increment $\Delta R = 15.8$ cm is to be added to each range observation.

Table 4. Difference between observed ranges, by use of centroids, and range to the LAGEOS center of mass. $\langle \Delta R_c \rangle$ is the average of N trails by using coherent light, and the rms column shows the variation of R_c for the N trials. ΔR_i is the calculated result for incoherent light pulses; it is the value to which $\langle \Delta R_c \rangle$ should converge.

Pulse length (nsec)	N	$\langle \Delta R_c \rangle$ (cm)	rms scatter (cm)	$\langle \Delta R_c \rangle - \Delta R_i$ (cm)
20	100	16.03	2.3	0.23
5	100	16.04	1.7	0.24
2	100	15.97	1.8	0.17
1	625	15.78	1.4	-0.02
0.2	100	15.65	0.7	-0.15

If care is taken to ensure that the cube corners are uniformly spaced over LAGEOS and that there are no systematic variations in, e.g., the cube-corner reflectivities over the surface of the sphere, then the largest excursions – viz., those resulting from variations in θ_{ij} – will surely be uncorrelated from one return to the next because of the great sensitivity of θ_{ij} to minute changes in satellite aspect angle.

We have also computed the pulse-to-pulse variation in retroreflected-pulse intensity. The results are shown in Figure 7. It can be seen that a considerable variation in return-pulse intensity is to be expected, in accord with our experience with the retroreflector satellites now in orbit. This wide variation in echo amplitude will result in some loss of returns because the laser receiving equipment has a limited dynamic range, which causes larger range errors at high and low signal levels. However, with a reasonable equipment dynamic range of 30:1, with a typical LAGEOS pass of 26-min duration (15° elevation limit), and with a 10-ppm rate, only 20 of 260 pulses would be lost. It is a straightforward matter to design circuitry to reject pulses below a minimum or above a maximum level.

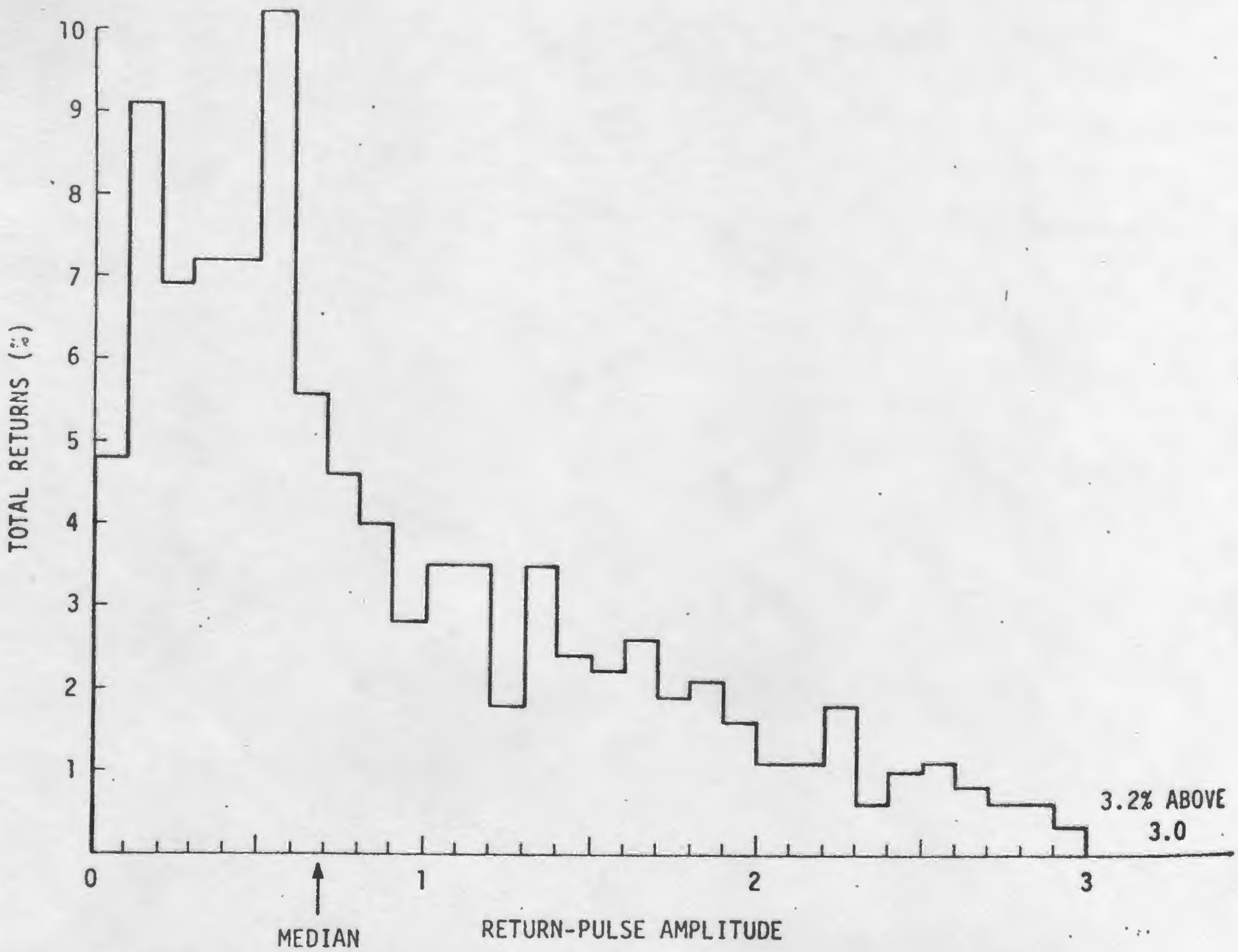

Figure 7. Computed probability distribution of the intensity of LAGEOS retroreflected pulses. The histogram is normalized so that the intensity of an incoherent pulse is unity on the horizontal scale.

We have calculated the intensity of the returns expected from LAGEOS, using the median value on the histogram. The assumed laser characteristics are those that can be expected of a good operational system at the time LAGEOS is launched:

Energy per pulse	1.5 J → 5 J.
Pulse width	1 nsec
Laser-beam radius	0.5 arcmin
Receiving-telescope diameter	50 cm
Photomultiplier-tube quantum efficiency	10%
Laser λ (ruby)	6943 Å

The results are shown in Table 5. The last column lists the fluctuations in pulse centroid contributed (solely) by the fact that the detected signal is comprised of a discrete number of photoelectrons (Lehr et al., 1970). Although the received signal strength can fluctuate to a considerable degree around the listed values, 50% of the returns being larger and 50% smaller, the number of photoelectrons per pulse taken on a statistical basis should be quite adequate. We conclude that the proposed retroreflector array is quite large enough to provide echo pulses of sufficient amplitude to support the LAGEOS mission.

Table 5. LAGEOS received signal levels in photons per pulse S and photoelectrons per pulse N, as a function of elevation angle. T is the atmospheric transmission factor, and the values are for "good" seeing conditions. The quantity E is the error in measuring a single pulse centroid caused by the "graininess" of the detected signal as a result of its being composed of N discrete electrons.

Elevation angle	Range (Mm)	T	S	N	E(N)
10°	6.80	0.19	60	6	2.6
15	6.35	0.32	230	23	1.3
20	5.95	0.42	520	52	0.90
25	5.59	0.49	900	90	0.67
30	5.27	0.55	1400	140	0.55
45	4.53	0.66	3800	380	0.34
60	4.06	0.71	6900	690	0.25
75	3.80	0.73	9500	950	0.21
90	3.72	0.74	10,000	1000	0.20