

Design of the LARES-2 Array

by David Arnold

Smithsonian Astrophysical Observatory, Cambridge, MA (retired)

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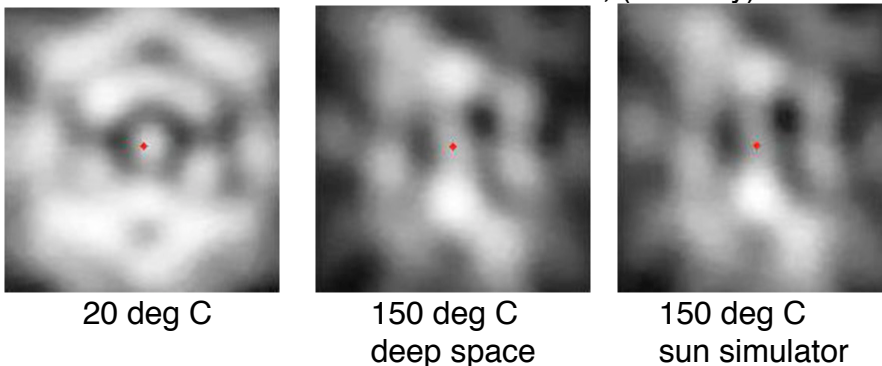
1. Abstract:

The design proposed for LARES-2 uses 1.0 inch uncoated COTS (Commercial Off-The-Shelf) cube corners with no intentional dihedral angle offset. The cubes are held in a floating mount that virtually eliminates conductive heat transfer to the cubes. The design minimizes the effect of thermal gradients and manufacturing errors in the dihedral angle offsets. The isothermal transfer function should be very close to the actual performance in orbit. Testing of a set of 10 COTS cubes shows good optical performance. The cubes are inexpensive and are manufactured in bulk. Simulations show that the systematic range errors should be on the order of a half millimeter.

2. Design goal:

The design goal for the GGOS program is one millimeter accuracy. The present geodetic satellites using 1.5 inch cube corners cannot provide this level of accuracy. Thermal effects are minimized by the use of uncoated cubes and a floating mount to prevent conductive heat transfer to the cube corners. However, thermal testing of the 1.5 inch LARES cubes shows significant residual thermal distortion of the diffraction pattern. The cubes provide sufficient cross section. But the thermal distortion causes unmodeled changes in the range correction. **The plots are full scale 110 μ rad.**

Test of LARES- Prism No. 51 , (in cavity)



D.Spano, PHD , La Sapienza Univ. di Roma, 2012

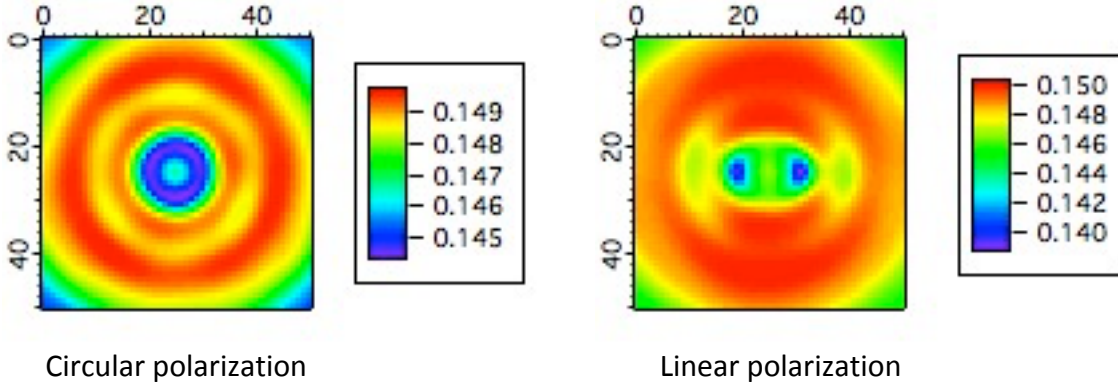
3. Polarization asymmetry.

In an uncoated cube there is an interaction between dihedral angle offsets and phase changes due to total internal reflection. This results in an asymmetrical diffraction pattern if linear polarization is used. The pattern has circular symmetry for circular polarization. The asymmetry can be virtually eliminated if no dihedral angle offset is used. The offset of 1.25 arcsec in the 1.5 inch cubes is necessary to account for velocity aberration. If 1.0 inch uncoated cubes are used the diffraction pattern is wide enough to account for velocity aberration without the need for a dihedral angle offset. **Lares-2 will be at the LAGEOS altitude, velocity aberration 32 - 40 μ rad.**

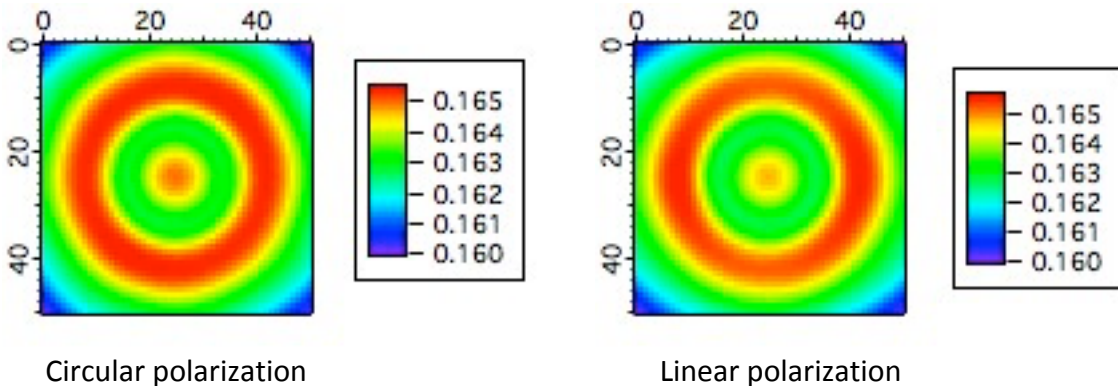
Simulations have been done comparing 1.5 and 1.0 inch uncoated cubes for LARES-2. The first design uses 204 1.5 inch cubes on a 200 mm radius satellite. The second design uses 303 1.0 inch cubes on a 202 mm radius satellite. The range correction and cross section matrices are irregular at a single incidence angle on the satellite for both circular and linear polarization. The matrices have been averaged over 2520 orientations of the satellite. When this is done a circular pattern shows up for circular polarization and an asymmetric pattern for linear polarization. The magnitude of the range correction is different for the two designs because the optical path length is different in the different size cubes and the radius of the satellite is slightly different. **The scale of the figures below is -50 to +50 μ rad.**

Centroid matrices

1.5 inch cubes



1.0 inch cubes

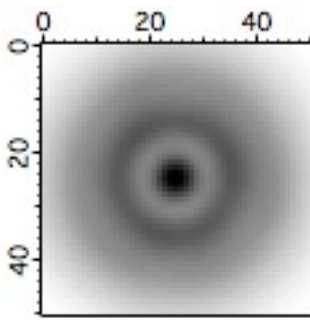


Circular polarization produces a circular pattern for both size cubes. The pattern is not completely circular for the 1.5 inch cubes even with averaging over many incidence angles. The pattern for 1.0 inch cubes has good circular symmetry.

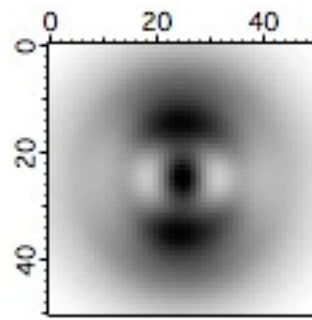
Linear polarization produces an asymmetric pattern for the 1.5 inch cubes with a dihedral angle offset of 1.25 arcsec. For the 1.0 inch cubes with no dihedral angle offset the pattern is nearly circular with linear polarization. There is a very small remaining asymmetry that is less than the accuracy goal for the satellite. **The scale is -50 to +50 μ rad.**

Cross section matrices

1.5 inch cube

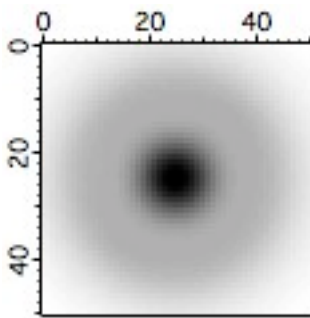


Circular polarization

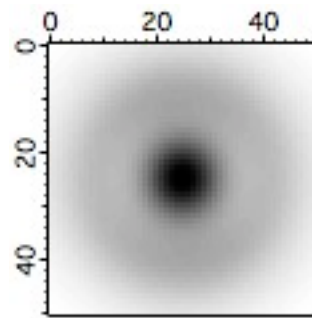


Linear polarization

1.0 inch cube



Circular polarization



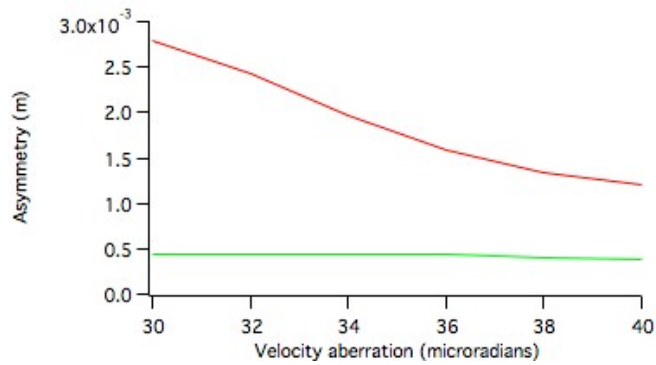
Linear polarization

The pattern with linear polarization is asymmetric for the 1.5 inch cubes but symmetric for the 1.0 inch cubes.

The maximum and minimum values of the centroid have been computed around circles of increasing radius in the far field. The asymmetry has been computed as the maximum minus the minimum. This difference has been plotted vs the magnitude of the velocity aberration.

Comparison of the asymmetry in the centroid

Red = 1.5 inch, Green = 1.0 inch



With the 1.0 inch cubes the asymmetry is less than .5 mm.

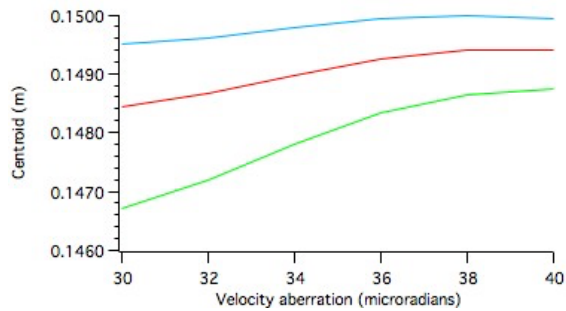
4. Centroid vs velocity aberration.

The proposed design minimizes the variation of the centroid with velocity aberration by choosing the size of the cube to place the velocity aberration on a peak in the pattern. The results are plotted in the figures below. The velocity aberration varies between about 32 and 40 microradians. Linear vertical polarization is used.

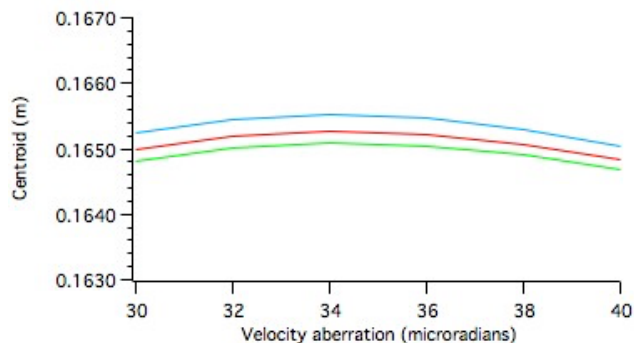
Centroid vs velocity aberration

1.5 inch cube

Red = average, Green = minimum, Blue = maximum



1.0 inch

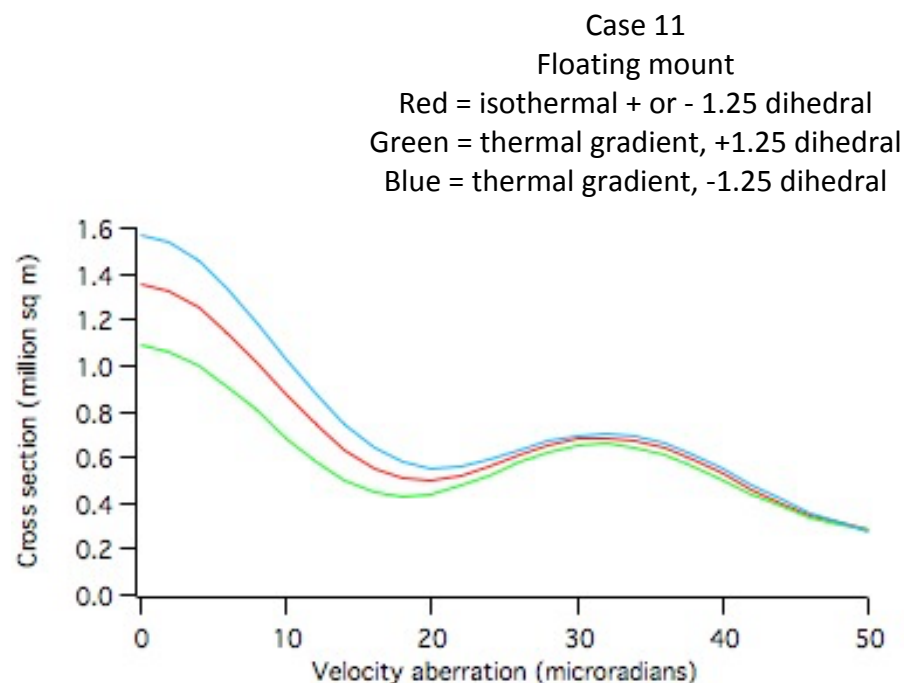


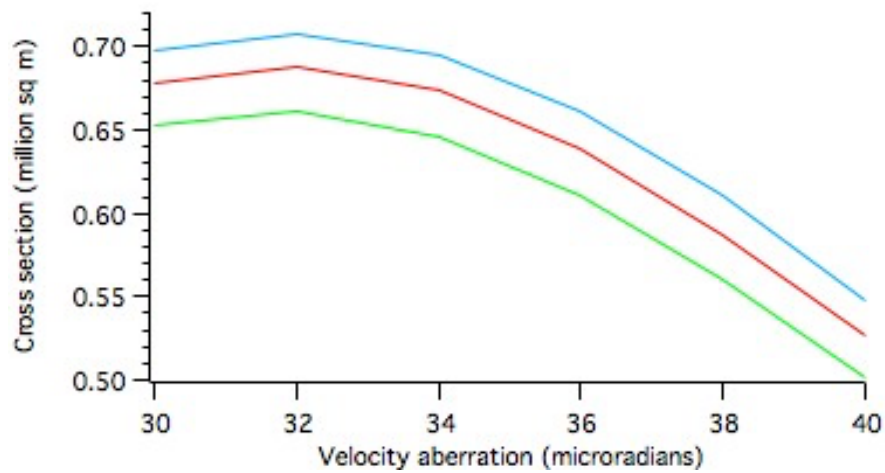
The average (red curve) for the 1.5 inch cube changes by .74 mm from 32 to 40 microradians. However, the asymmetry of the pattern causes significant range errors. The change of the red curve is .47 mm for the 1.0 inch cube with very little asymmetry. This is within the accuracy goal of one millimeter. A correction could be applied as a function of velocity aberration.

5. Thermal simulations

Two thermal simulations have been done in Italy to compare a floating mount (Case 11) with one where there is strong conductive contact (Case 12) between the mount and the cube corner. The output of the thermal simulations is a three-dimensional matrix giving the temperature distribution in the cube. I have used the temperature matrix to compute the diffraction pattern under various conditions of polarization and dihedral angles. The first step is to do a ray tracing to get the phase front due to the thermal gradients. The next step is to add phase due to dihedral angle offsets, total internal reflection, and polarization of the incident wave. Three simulations have been run for each case. The reference case is with a dihedral angle offset of 1.25 arcsec with no thermal gradient. With no thermal gradient the cross section vs velocity aberration is the same for either +1.25 or -1.25 arcsec. The phase front due to a thermal gradient may be either primarily concave or primarily convex. The phase front for a dihedral angle offset is either concave or convex depending on the sign. A dihedral angle offset can either partially cancel the effect of a thermal gradient or add to it.

In the figure below for Case 11 the cross section is averaged around circles of increasing radius in the far field and plotted vs the magnitude of the velocity aberration. **The simulations are done for a single cube corner.**



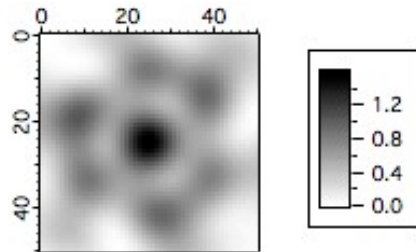


Curve microrad cross section
 Blue 32.000000 0.707174
 Red 32.000000 0.687217
 Green 32.000000 0.660673

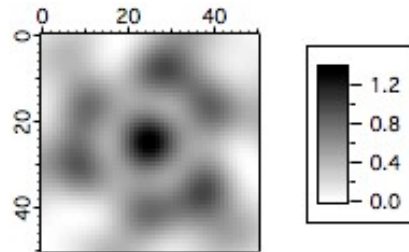
The ratio of the maximum and minimum cross section at 32 microradians is Blue/Green = 1.07.
 This is a very small change.

Diffraction patterns for the three curves in the figure above.

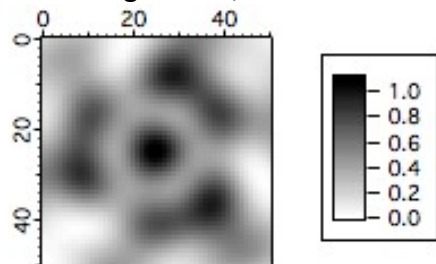
Thermal gradient, Dihedral -1.25



Isothermal, Dihedral 1.25



Thermal gradient, Dihedral +1.25



The diffraction patterns do not show large differences. The central intensity is larger for -1.25 arcsec and smaller for +1.25 arcsec. **The scale is -50 to +50 μ rad.**

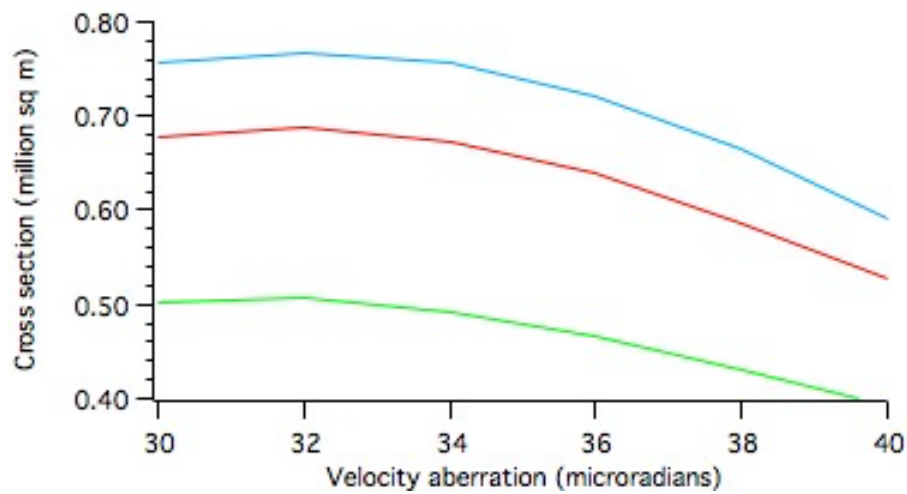
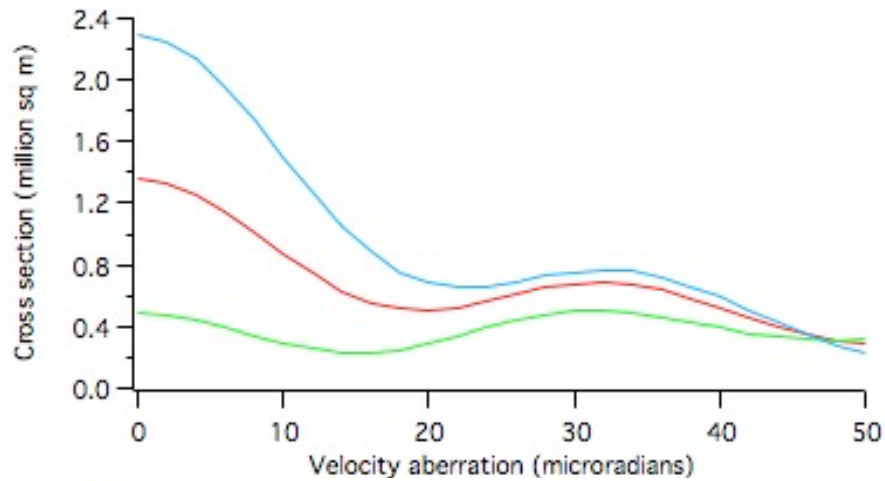
Case 12

Pressure on the cube

Red = isothermal + or - 1.25 dihedral

Green = thermal gradient, +1.25 dihedral

Blue = thermal gradient, -1.25 dihedral



Curve microrad cross section

Blue 32.000000 0.768003

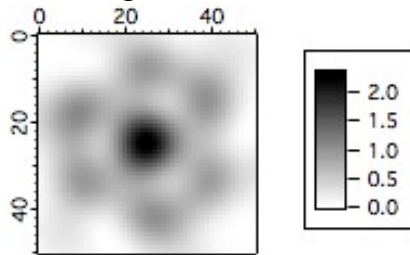
Red 32.000000 0.687217

Green 32.000000 0.505958

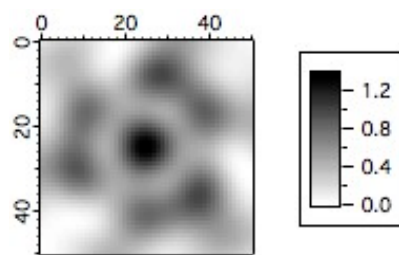
The ratio between the maximum and minimum cross section at 32 microradians is Blue/Green = 1.52. The fractional change is 7.5 times greater than with the floating mount (Case 11).

Diffraction patterns for the three cases

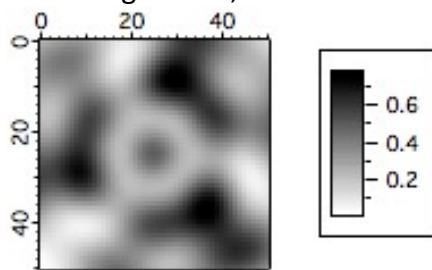
Thermal gradient, dihedral -1.25



Isothermal, dihedral + or - 1.25



Thermal gradient, dihedral +1.25



The patterns show significant effects due to the thermal gradient particularly in the central intensity. The central peak is never observed. However, the changes in the velocity aberration range from 32 to 40 microradians are significant. **The scale is -50 to +50 μ rad.**

6. Summary.

The use of small cubes eliminates the need for dihedral angle offsets. This allows the use of inexpensive COTS cubes. The small cubes produce a much more accurate isothermal range correction. The isothermal range correction will be very close to the actual range correction in orbit since the small cubes significantly reduce the effect of thermal gradients.

Specifically, the benefits are:

1. There is more uniform coverage of the surface with smaller variations with incidence angle.
2. The 1.5 inch cubes are too large for the velocity aberration and required dihedral angle offsets. This produces a "lumpy" diffraction pattern that causes variations in range within the far field diffraction pattern.

- 3.** There is an interaction between dihedral angle offsets and the phase changes due to total internal reflection that produces an asymmetrical diffraction pattern when linear polarization is used.
- 4.** The 1.0 inch cubes provide the necessary beam spread to account for velocity aberration without the need for dihedral angle offsets. This also removes the asymmetry in the diffraction pattern with linear polarization.
- 5.** The diffraction pattern without dihedral angle offsets is smoother than the patterns with offsets.
- 6.** The diffraction pattern of an uncoated cube has a ring of spots around the central peak. The size of the cube can be chosen to put the velocity aberration on this ring of spots rather than on a slope in the diffraction pattern. This reduces the variation of the range correction with velocity aberration. This ring of spots is a very stable part of the diffraction pattern that does not change much due to various perturbations.
- 7.** Thermal effects increase as some power (approximately the 4th power) of the size. The reduction in size from 1.5 to 1.0 inches appears to reduce variations in the cross section by about a factor of 5 or 6.
- 8.** Eliminating the dihedral angle offset makes it possible to use COTS (Commercial Off-The-Shelf) cubes that are inexpensive and available quickly. Testing by Ludwig Grunwaldt shows that the optical quality of these cubes is as good as custom made cubes with dihedral angle offsets that are expensive and time consuming to manufacture.
- 9.** There are small unintentional dihedral angle offsets in COTS cubes that are generally less than one arcsec but can be up to two arcsec. The effect of a positive ($>90^\circ$) offset is in the opposite direction from the effect of a negative ($<90^\circ$) offset. Since the mean offset is zero the positive offsets tend to partially cancel the effect of the negative offsets.
- 10.** Thermal simulations show that the effect of thermal gradients in a 1.0 inch cube is very small with a floating mount.
- 11.** A floating mount requires leaving a small gap between the ring and the cube. This could potentially result in damage to the cube due to vibrations during launch. Vibration testing with a very large gap showed no damage to the cube.